Second, But Not Last: Competition with Positive Spillovers

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Research Article

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Abstract: This paper extends the traditional rent-seeking model to consider contests in which the effects of the contestants’ efforts are externally unproductive (i.e., redistributive) but internally productive (i.e., with positive spillover effects on other contestants). Our results show that when players act sequentially, the presence of positive spillovers on other contestants may reduce, or even reverse, the first-mover’s advantage. A second-mover advantage is very likely to arise. Notably, in contests with multiple players, the second-mover advantage does not unravel into a last-mover advantage. Players want to be second, but not last. The comparative statics analysis shows how the strength of positive spillovers affects contestants’ equilibrium expenditures and payoffs, and aggregate rent dissipation.

Keywords: competition; sequential rent seeking; spillover effects; second-mover advantage

JEL Classification: C72; D72

1 Introduction

In the original formulation of rent-seeking contests, two or more players simultaneously expend resources to win a prize, and their efforts increase their respective probabilities of success, decreasing that of their competitors (e.g., Tullock 1967, 1980; An earlier version of this paper was circulated under the title “Sequential Rent Seeking with Positive Spillover Effects.”

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This formulation of the rent-seeking contests obscures several factors that are often present in real-life contests. One such factor is that contests often take place sequentially and the actions of past contestants have spillover effects on subsequent movers. Sequential rent-seeking contests with negative spillover effects have been previously analyzed in the literature (e.g., Dixit 1987; Glazer and Hassin 2000; Leininger and Yang 1994; Morgan 2003; Stracke 2013). The two general findings are the following: under sequential rent-seeking games, the first-mover has a substantial advantage over the other players, earning higher payoffs than subsequent movers; and aggregate rent-seeking expenditures are larger than in simultaneous games.

In this paper, we investigate the mirror-image case of positive spillover effects. Situations of rent-seeking with positive spillover effects, in which players benefit from prior contestants’ efforts, are ubiquitous in real life. Positive spillovers are generally present in political campaigns. Candidates competing for a presidential nomination, put forth arguments that may pave the road to subsequent entrants in a primary race, helping their opponents’ political chances. In political lobbying, competing groups with interrelated political agendas create positive spillovers, helping one another’s mission by persuading lawmakers and regulators about the merits of their political goals. Positive spillovers are observed in many other common activities. In sports races, the observed performance of early contestants can provide information about the race track conditions or the snow conditions of a ski run to subsequent racers. Likewise, in bicycle and car racing, the first racer in a group provides a positive spillover to followers, allowing them to engage in slipstreaming and drafting. In music contests, the first contestant may help the ratings of subsequent performers by warming up the audience. In patent races and other R&D contests, the early research efforts of one firm often generate informational spillovers that can increase the odds of success for subsequent competitors.

In this paper, we explore how the presence of positive spillovers interacts with the first-mover advantage and the equilibrium investments observed in the other settings examined in the literature. Specifically, we analyze the equilibrium efforts and payoffs in simultaneous Cournot games (Section 2), sequential Stackelberg games (Section 3), and semi-sequential Dixit games (Section 4). The results show that positive spillovers in both sequential and semi-sequential contests lead to a robust second-mover advantage and reduce aggregate rent-seeking expenditures. Interestingly, in

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1 Following Tullock (1967, 1980), rent-seeking models have been used to analyze competition in a wide variety of contexts, including political lobbying, all-pay auctions, and patent races. See Baye and Hoppe (2003) and Congleton, Hillman, and Konrad (2008a,b).

2 Münster (2009), Rai and Sarin (2009) and Chowdhury and Sheremeta (2011a,b) analyzed simultaneous rent-seeking games with positive spillovers within groups of competitors. See also Baik and Lee (2000).
contests with multiple players, the second-mover advantage does not unravel into a last-mover advantage. Players want to be second, but not last. In Section 5, we conclude, with some discussion of policy implications of our results, and ideas for future extensions.

2 The Cournot Model

We consider three players, \( i \in \{1, 2, 3\} \), with the same valuation for the rent, i.e., \( V = 1 \). Let \( y_{c,i} \), \( i \in \{1, 2, 3\} \) denote the effort levels, which are non-negative real numbers. Players simultaneously decide their efforts. Each player’s effort may produce positive spillovers on the others’ winning probability. Let \( \alpha \in [0, 1] \) denote positive spillovers.

Each player has a linear production function for effective effort. The marginal rate of technical substitution among the individual effort and positive spillover effect does not vary across players. Following Münster (2009) and Rai and Sarin (2009), players’ payoffs are:

\[
\pi_{c,i} = \frac{y_{c,i} + a(y_{c,j} + y_{c,z})}{(1 + 2\alpha)(\sum_{i=1}^{3} y_{c,i})} - y_{c,i} \quad i, j, z \in \{1, 2, 3\}, \ i \neq j \neq z
\]

(2.1)

Each player invests in effort to maximize the expected payoff, subject to the participation constraints that the expected payoff is non-negative, i.e., \( \pi_{c,i} \geq 0 \). Let \( y_{c,i}^{*}, \ i \in \{1, 2, 3\} \), denote the optimal effort. In equilibrium, \( y_{c,1}^{*} = y_{c,2}^{*} = y_{c,3}^{*} = y_{c}^{*} \), where \( y_{c}^{*} \) is given by:

\[
y_{c}^{*} = \frac{2(1 - \alpha)}{9(1 + 2\alpha)}
\]

(2.2)

Each player’s optimal payoff is given by:

\[
\pi_{c}^{*} = \frac{1 + 8\alpha}{9(1 + 2\alpha)}
\]

(2.3)

The aggregate effort \( Y_{c}^{*} = 3 y_{c}^{*} \) is equal to \( 2(1 - \alpha)/3(1 + 2\alpha) \), and the aggregate payoff \( \Pi_{c}^{*} = 3 \pi_{c}^{*} \) is equal to \( 1 + 8\alpha/3(1 + 2\alpha) \).

Result 2.1. (Cournot Equilibrium). As positive spillovers increase, equilibrium efforts decrease and equilibrium payoffs increase.

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3 In the literature, the function that maps efforts into winning probability is commonly referred to as the contest success function (CSF). For an axiomatic justification of Tullock’s CSF, see Skaperdas and Grofman (1995); Kooreman and Schoonbeek (1997); Clark and Riis (1998).
Result 2.2. (Cournot Participation). All the players exert positive effort for $\alpha \in [0, 1)$. For $\alpha = 1$, players exert zero effort.

Result 2.3. (Cournot Rent Dissipation). Rent dissipation decreases as positive spillovers increase. Full rent-dissipation never occurs.

3 The Stackelberg Model

Let us now consider Player 1 moving first, Player 2 moving second, and Player 3 moving last. Players’ payoffs are:

\[
\pi_{s,1} = \frac{y_{s,1}}{y_{s,3} + (1 + \alpha)(y_{s,2} + (1 + \alpha)y_{s,1})} - y_{s,1}
\]

(3.1)

\[
\pi_{s,2} = \frac{y_{s,2} + ay_{s,1}}{y_{s,3} + (1 + \alpha)(y_{s,2} + (1 + \alpha)y_{s,1})} - y_{s,2}
\]

(3.2)

\[
\pi_{s,3} = \frac{y_{s,3} + ay_{s,2} + (1 + \alpha)y_{s,1}}{y_{s,3} + (1 + \alpha)(y_{s,2} + (1 + \alpha)y_{s,1})} - y_{s,3}
\]

(3.3)

The model is solved by backward induction. Player 3 observes the aggregate spending of players 1 and 2, $g_2 = y_{s,1} + y_{s,2} + ay_{s,1}$. Player 3’s expected payoff is given by:

\[
\pi_{s,3} = \frac{y_{s,3} + ag_2}{y_{s,3} + (1 + \alpha)g_2} - y_{s,3}
\]

(3.4)

Player 3’s optimal effort is:

\[
y_{s,3}^* = \sqrt{g_2} - g_2(1 + \alpha)
\]

(3.5)

Player 2 observes $y_{s,1}$ and treats it as fixed, whereas he observes $y_{s,3}$ as a function of $y_{s,2}$. Player 2 maximizes the following function with respect to $y_{s,2}$:

\[
\pi_{s,2} = \frac{y_{s,2} + ay_{s,1}}{y_{s,3}(y_{s,2}) + (1 + \alpha)g_2} - y_{s,2} = \frac{y_{s,2} + ay_{s,1}}{\sqrt{g_2} - g_2} - y_{s,2}
\]

(3.6)

Player 2’s optimal effort is:

\[
y_{s,2}^* = g_2\left(a + 2(1 - (1 + \alpha)\sqrt{g_2})\right)
\]

(3.7)

Given $y_{s,1} = g_2 - y_{s,2}^*/(1 + \alpha)$, Player 1 maximizes his expected payoff subject to (3.7):

\[
\pi_{s,1} = 3g_2 - 2g_2^{3/2} - \sqrt{g_2}
\]

(3.8)
We obtain $g_2^* = \frac{1}{6} (2 \pm \sqrt{3})$. We consider $g_2^* = 0.622$, since the other solution generates a negative value for $y_{s,1}^*$. The optimal efforts are:

$$y_{s,1}^* = 0.359$$ (3.9)

$$y_{s,2}^* = 0.262 - 0.359\alpha$$ (3.10)

$$y_{s,3}^* = 0.166 - 0.622\alpha$$ (3.11)

Each player’s optimal payoffs – denoted by $\pi_{s,1}^*$, $\pi_{s,2}^*$, $\pi_{s,3}^*$ – are obtained by substituting (3.9), (3.10), (3.11) respectively into (3.1), (3.2), (3.3). For $\alpha > 0.267$, Player 3’s optimal effort becomes negative, and a proper equilibrium no longer exists. The aggregate effort $Y_s^* = y_{s,1}^* + y_{s,2}^* + y_{s,3}^*$ is equal to $0.788 - 0.981\alpha$, and the aggregate payoff $\Pi_s^*$ is $\pi_{s,1}^* + \pi_{s,2}^* + \pi_{s,3}^*$.

**Result 3.1.** (Stackelberg Equilibrium). Late-movers’ optimal effort decreases as positive spillovers increase. As positive spillovers increase, the second mover earns higher payoffs than the other players. The first mover makes higher payoffs than late-movers only for $\alpha < 0.087$.

**Result 3.2.** (Stackelberg Participation). For $\alpha \in [0, 0.267)$, players exert positive effort. For $\alpha \in [0.267, 1]$, the last mover exerts zero effort.

**Result 3.3.** (Stackelberg Rent Dissipation). Rent-dissipation decreases as positive spillovers increase. For $\alpha \in [0, 0.267)$, rent-dissipation is higher in the Stackelberg game than in the Cournot game.

### 4 The Second-Mover Advantage

An interesting result of our analysis is that positive spillovers lead to a robust second-mover advantage. Interestingly, in contests with multiple players, the second-mover advantage does not unravel into a last-mover advantage. Players want to be second, but not last.

To show this, consider the four necessary conditions for the existence of a first-mover advantage (Glazer and Hassin 2000): (i) the first-mover should gain higher payoffs than later-movers (i.e., he would not want to delay his move); (ii) the first-mover should gain higher payoffs than in a Cournot equilibrium (i.e., if given an opportunity, he would not want to move simultaneously with another player); (iii) a player with the opportunity to move second should prefer to move second than to
move later, and so on (i.e., the advantage is monotonic and not double-picked); and (iv) the advantages of moving first are robust to changes in the sequence of later entrants (i.e., the first-mover is better off, whether later-movers play simultaneously or sequentially).

In our framework, the first condition is rarely satisfied: for \( \alpha > 0.087 \), the first-mover’s payoffs fall below the other players’ payoffs. The second condition is never satisfied: in a Stackelberg game, the first-mover’s payoffs are never higher than in a Cournot game (Table 2). With respect to the third condition, players always prefer to move second rather than later. The second-mover’s payoffs are 0.123 when \( \alpha = 0.1 \); 0.183 when \( \alpha = 0.2 \); 0.227 when \( \alpha = 0.267 \), which are all higher than the third mover’s payoffs (i.e., 0.113, 0.174, 0.21, respectively).

To evaluate the fourth condition, we need to consider a Dixit game in which one player moves first, and the remaining players move simultaneously, playing a Cournot game.4 Let Players 2 and 3 – that play à la Cournot – observe \( y_{d,1} \) and treat it as fixed. Their expected payoffs are:

\[
\pi_{d,2} = \frac{y_{d,2} + \alpha y_{d,1}}{y_{d,3} + (1 + \alpha)(y_{d,1} + y_{d,2} + \alpha y_{d,1})} - y_{d,2} \tag{4.1}
\]

\[
\pi_{d,3} = \frac{y_{d,3} + \alpha (y_{d,1} + y_{d,2} + \alpha y_{d,1})}{y_{d,3} + (1 + \alpha)(y_{d,1} + y_{d,2} + \alpha y_{d,1})} - y_{d,3} \tag{4.2}
\]

We obtain \( y_{d,2}^* = y_{d,3}^* = y_{d,23}^* \), where \( y_{d,23}^* \) is given by:\(^5\)

\[
y_{d,23}^* = 1 - \frac{2y1(1+\alpha)^2(2+\alpha) + \sqrt{1+4y1(1+\alpha)(2+\alpha)}}{2(2+\alpha)^2} \tag{4.3}
\]

Player 1’s expected payoff is given by:

\[
\pi_{d,1} = \frac{y_{d,1}}{y_{d,23}^* + (1 + \alpha)(y_{d,1} + y_{d,23}^* + \alpha y_{d,1})} - y_{d,1} \tag{4.4}
\]

Player 1’s optimal effort is given by:

\[
y_{d,1}^* = \frac{3 + \alpha}{4(\alpha + 2)} \tag{4.5}
\]

Players 2 and 3’s optimal efforts can be rewritten as:

---

4 Dixit (1987) found that the first-mover’s payoffs are higher than a player’s payoffs in a Cournot equilibrium.
5 The other solution to the maximization problem, i.e., \( y_{d,23}^* = \frac{-1+2y1(1+\alpha)^2(2+\alpha)+\sqrt{1+4y1(1+\alpha)(2+\alpha)}}{2(2+\alpha)^2} \) is negative for all values of \( \alpha \) and \( y_1 \).
Table 1: Aggregate payoffs.

<table>
<thead>
<tr>
<th>Game/α</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.267</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>0.210</td>
<td>0.356</td>
<td>0.509</td>
<td>0.616</td>
<td>0.671</td>
<td>0.848</td>
<td>1.046</td>
<td>1.283</td>
<td>1.592</td>
<td>2.053</td>
<td>2.945</td>
<td>6.290</td>
</tr>
<tr>
<td>Cournot</td>
<td>0.333</td>
<td>0.498</td>
<td>0.619</td>
<td>0.681</td>
<td>0.708</td>
<td>0.778</td>
<td>0.833</td>
<td>0.879</td>
<td>0.917</td>
<td>0.949</td>
<td>0.976</td>
<td>1.000</td>
</tr>
<tr>
<td>Dixit</td>
<td>0.250</td>
<td>0.353</td>
<td>0.451</td>
<td>0.514</td>
<td>0.545</td>
<td>0.634</td>
<td>0.720</td>
<td>0.803</td>
<td>0.883</td>
<td>0.961</td>
<td>1.037</td>
<td>1.111</td>
</tr>
</tbody>
</table>
The payoffs in equilibrium are obtained substituting (4.5) and (4.6) into (4.4), (4.1), (4.2). The aggregate effort and payoff are defined as $Y_d^* = y_{d,1}^* + y_{d,2}^* + y_{d,3}^*$ and $\Pi_d^* = \pi_{d,1}^* + \pi_{d,2}^* + \pi_{d,3}^*$. Players 2 and 3’s optimal efforts become negative for $\alpha > 0.414$. Thus, Dixit equilibrium exists for $\alpha \in [0, 0.414]$. The participation range is higher in a Stackelberg game – where Player 3 participates if $\alpha > 0.267$ – rather than in a Dixit game – where Player 3 participates if $\alpha > 0.414$. The presence of positive spillovers generates a second-mover advantage.

**Result 4.1. (Second-Mover Advantage).** In a Stackelberg game, the second-mover’s payoff is always higher than the third-mover’s payoff. For $\alpha > 0.087$, the first-mover’s payoff is always lower than all the other players’ payoffs.

**Result 4.2. (First-Mover Disadvantage).** The first-mover’s payoff is higher in a Cournot equilibrium than in a Stackelberg equilibrium for all values of $\alpha$. The first-mover’s payoff is higher in a Cournot game than in a Dixit game, for $\alpha > 0.024$.

Figure 1 shows a graphical representation of the first mover’s effort and payoffs in the three games, with corresponding numerical values reported in Table 2. Table 1 reports the aggregate payoffs in the three games.
### Table 2: First-mover’s payoffs.

<table>
<thead>
<tr>
<th>Game/(\alpha)</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.267</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>0.096</td>
<td>0.120</td>
<td>0.152</td>
<td>0.179</td>
<td>0.194</td>
<td>0.252</td>
<td>0.332</td>
<td>0.449</td>
<td>0.634</td>
<td>0.963</td>
<td>1.697</td>
<td>4.738</td>
</tr>
<tr>
<td>Cournot</td>
<td>0.111</td>
<td>0.166</td>
<td>0.206</td>
<td>0.227</td>
<td>0.236</td>
<td>0.259</td>
<td>0.278</td>
<td>0.293</td>
<td>0.306</td>
<td>0.316</td>
<td>0.325</td>
<td>0.333</td>
</tr>
<tr>
<td>Dixit</td>
<td>0.125</td>
<td>0.131</td>
<td>0.136</td>
<td>0.140</td>
<td>0.141</td>
<td>0.146</td>
<td>0.150</td>
<td>0.154</td>
<td>0.157</td>
<td>0.161</td>
<td>0.164</td>
<td>0.167</td>
</tr>
</tbody>
</table>
5 Moving Forward

Conventional theories of competition classify contests as being either “productive” or “unproductive,” with rent-seeking falling in the latter category. As previously pointed out by Guerra, Luppi, and Parisi (2019), this dichotomous classification of real-world situations fails to recognize the entire spectrum of competitive activities. Building on this premise, this paper extends the traditional analysis to consider contests in which the effects of the contestants’ efforts are externally unproductive (i.e., the competitive efforts are merely redistributive and do not generate a surplus for society), but internally productive (i.e., they create a positive spillover effect that facilitates the competitive opportunities of other players). Our results show that when players act sequentially and their efforts have positive spillover effects on the other contestants, a second-mover advantage is very likely to arise. Aggregate spending is higher in sequential settings and, if given an option under a veil of sequence-uncertainty, contestants would prefer to compete simultaneously rather than sequentially. Future analytical extensions of our work should consider the effect of positive spillovers in rent-seeking contests with a larger number of players and non-linear CSF, contestants with asymmetric strengths, and situations where the magnitude of the spillovers can be endogenously controlled by the contestants at a cost.

Our findings help explain several real-life situations where contestants who appear to be ready to lead a race may slightly delay their entry, but not indefinitely. As previously pointed out by Reinganum (1985), in some situations, it is more profitable to enter a competitive contest late. Our findings complement these findings, explaining why, in the presence of positive spillovers, contestants may delay their entry into the race but they would do so only slightly, to let only one of their competitors move ahead of them. The implications of these findings hinge upon the nature of the rent that contestants are competing for. As discussed in the literature (e.g., Chung 1996; Congleton 1989; Paul and Wilhite 1994; Lee and Kang 1998), rent-seeking activities may generate positive or negative externalities on society as a whole, and the effects observed in our model may thus be viewed as desirable or undesirable depending on the relationship between the private value and the social value of the competitive activity.

Political campaigns often attempt to shift the preferences of the median voter on a given policy issue. Early movers are likely to find greater voters’ resistance, given the current views of the population. As the campaign progresses and early movers pour resources into their campaign, the views of the public may gradually shift. A few voters may accept the values espoused by the early campaigner, but more voters may join along as the campaign progresses. In this way, the first-mover may fail its political mission, but by doing so it would pave the road for the subsequent mover’s
campaign. For example, candidates competing for a presidential nomination put forth arguments that may pave the road to subsequent entrants in a primary race, helping their opponents’ political chances. Competing groups with interrelated political agendas create positive spillovers, helping one another’s mission by persuading lawmakers and regulators about the merits of their political goals. Similarly, great political figures are often too early for their time. The reason why their untimely political thoughts are not understood and not well received is that the world simply is not yet acquainted with their way of thinking and the ideas that they are trying to bring to society. Yet, their efforts opened new doors, and other second-movers can reap the political opportunities that they created. In all these examples, the second mover has an advantage because of the positive spillovers created by his predecessor. However, the second-mover advantage does not unravel into a last-mover advantage for the other contestants. Waiting too long may entail losing the opportunity to lead a winning campaign or to reap the benefits of a new political vision.

An important area of application of our findings is that of patent races and other R&D contests. In these research contests, the early investments of one firm often generate informational spillovers that can increase the odds of success for subsequent entrants. For example, in a patent race, the information created by unsuccessful research carried out by a first-mover can benefit subsequent competing firms. Failed research attempts generate useful information for other researchers because it prevents them from pursuing the same fruitless lines of inquiry. Whenever competing firms can obtain an informational advantage from the research of earlier movers, a positive spillover occurs (Guerra, Parisi, and Sawicki 2023). Our results suggest that these spillover effects may lead firms to hold out of the race, waiting for some other firm to make the initial investment. As a result, we may observe a lag in the beginning of research and the discovery process may progress at a slower pace than is socially desirable.

These examples provide ample opportunity for empirical investigation. Empirical work can be carried out to explore situations where early movers create positive spillovers on subsequent entrants. These studies should verify whether the timing of entry and the rate of success of first-movers and second-movers in political or market settings varies when early contestants create positive spillovers on subsequent entrants’ chances of success. Our findings further provide testable hypotheses for experimental evaluations (e.g., Zizzo 2002).

References


