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SOLVING A PROFOUND FLAW IN FRAUD-ON-THE-MARKET THEORY: UTILIZING A DERIVATIVE OF ARBITRAGE PRICING THEORY TO MEASURE RULE 10b-5 DAMAGES

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INTRODUCTION

The idea of a free, open and well-developed securities market is premised on the hypothesis that the competing judgments of buyers and sellers regarding a security's fair price will drive market prices to reflect, as much as possible, that security's just price.¹ Investors who buy and sell stock at market prices do so in reliance on the integrity of that price.² In fact, "it is hard to imagine that there ever is a buyer or seller who does not rely on market integrity. Who would knowingly roll the dice in a crooked crap game?"³ Because of this reliance on the integrity of market prices, when materially misleading statements are disseminated into the market, such misleading information affects the price of certain securities and, thereby, affects all those who trade in those securities.⁴ In a Rule 10b-5 action, once a plaintiff establishes there is liability for materially misleading statements that affected the securities market, the issue of damages arises. Without knowing the exact extent of an investor's reliance on a security's price, or the exact extent misleading information affected a security's price, it is difficult to ascertain with any degree of certainty the amount of damages caused. In general, damages in Rule 10b-5 cases are measured as the "difference between the price [of the security] under correct information and the actual market price."⁵

The practice of measuring damages in Rule 10b-5 suits came to rely on the capital market theory known as the Efficient Capital Markets Hypothesis ("ECMH") long before the United States Supreme Court's implied acceptance of it in *Basic Inc. v. Levinson*.⁶ The ECMH holds that securities' prices adjust quickly and without bias to publicly available information. Assuming that the ECMH is valid, it is but a short step from the application of other capital market theories to the calculation of damages in Rule 10b-5 suits.

The celebrated capital market theory that has been applied to the measurement of damages in Rule 10b-5 suits is the market model, which is an empirical derivation of the Capital Asset-Pricing Model

¹ See *Basic Inc. v. Levinson*, 485 U.S. 224, 246 (1988) (citation omitted).

² See *id.*

³ *Id.* at 246-47 (quoting *Schlanger v. Four-Phase Sys., Inc.*, 555 F. Supp. 535, 538 (S.D.N.Y. 1982)).

⁴ See *id.* at 247.

⁵ RONALD J. GILSON & BERNARD S. BLACK, *THE LAW AND FINANCE OF CORPORATE ACQUISITIONS* 138 (Supp. 1993).

⁶ 485 U.S. 224 (1988).

("CAPM").⁷ The market model holds that a security's return is a linear function of a general market factor, where the general market factor serves as a proxy of economic conditions that influence returns to securities in the capital market. The CAPM is an equilibrium asset-pricing theory which posits that the sole variable that explains the differences in expected returns for securities is a risk coefficient known as beta (β). Beta is the ratio of the covariance of a security's return and the market's return to the variance of the market's return, $(s_{sm} \div s_m^2)$.⁸ The CAPM asserts that the relationship between a security's expected returns and beta is positive and linear.

This Article contends that, contrary to its present use in the securities fraud realm as sanctioned by the Supreme Court and as assumed to be correct by most commentators, the CAPM is irrelevant for measuring damages in Rule 10b-5 cases because the CAPM is not designed to measure what stock prices would have been if the requisite fraud had not occurred. Instead, the CAPM is designed to help investors and portfolio managers determine optimal asset portfolios. That is, the CAPM is designed to help people make decisions about assets with uncertain *future* returns, rather than to analyze the actual *past* returns of assets in a manner that would allow for the measurement of damages pursuant to Rule 10b-5.

This Article further asserts that the legal community should, in the context of the measurement of damages in Rule 10b-5 cases, reject the use of empirical derivations of the CAPM, and, in its place, use more accurate models designed to analyze past asset performance. In attacking the very foundations of the present methodology for calculating securities fraud damages, Part I of this Article reviews Rule 10b-5 and the basic methodology of calculating damages in Rule 10b-5 cases. Part II examines the intersection of judicial theory and capital market theory by considering the fraud-on-the-market doctrine and the capital market theories under which the doctrine and the measurement of damages may be justified: the Efficient Capital Markets Hypothesis, systematic risk compensation and the Capital As-

⁷ The CAPM was developed in the 1960s by a number of financial economists in response to the groundbreaking work of Harry Markowitz regarding portfolio theory. See Harry Markowitz, *Portfolio Selection*, 7 J. FIN. 77 (1952). Most notable among the developers of the CAPM were Sharpe, Lintner and Mossin. See John Lintner, *The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets*, 47 REV. ECON. STAT. 13 (1965); Jan Mossin, *Equilibrium in a Capital Asset Market*, 34 ECONOMETRICA 768 (1966); William F. Sharpe, *Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*, 19 J. FIN. 425 (1964).

⁸ This measure of risk is commonly referred to as "beta" (β).

set-Pricing Model. Part III contrasts the forward-looking basis of the CAPM with the historical basis for Rule 10b-5 damages. As Part III illustrates, whereas the CAPM approach calculates the historical relationship between stock price and the Standard & Poor's 500 Index ("S&P 500 Index" or "S&P 500") and then projects the relationship forward to the period of the fraud, the technique we employ recognizes that it is erroneous to discount or ignore other factors besides movements in the S&P 500 Index that might actually have an even more powerful causal impact on a relevant security's price. Part IV introduces an alternative to the CAPM for damage measurement, utilizing the principles of arbitrage pricing theory. Finally, Part V surveys an empirical study that compares the ability of the CAPM with other derivative methods to explain the returns of common stocks and also introduces our own empirical study—one that is clearly superior to the CAPM's derivatives for measuring Rule 10b-5 damages.

I. RULE 10b-5 AND THE CALCULATION OF DAMAGES

A. *Rule 10b-5 Overview*

Rule 10b-5 is one of the most significant remedies for fraud provided under the Securities Exchange Act of 1934.⁹ It is the primary antifraud weapon against material misrepresentations or nondisclosures by issuers, insider trading and corporate mismanagement involving securities transactions. The Securities Exchange Commission ("SEC") promulgated Rule 10b-5 under the rulemaking authority vested in it by Congress under § 10 of the Securities Exchange Act of 1934. Unchanged since its promulgation in 1942, Rule 10b-5 provides:

It shall be unlawful for any person, directly or indirectly, by the use of any means or instrumentality of interstate commerce, or of the mails or of any facility of any national securities exchange,

- (a) To employ any device, scheme, or artifice to defraud,
- (b) To make any untrue statement of a material fact or to omit to state a material fact necessary in order to make statements made, in the light of the circumstances under which they were made, not misleading, or

⁹ It is estimated that at least one-third of the actions brought under the federal securities statutes involve Rule 10b-5. See 1 ALAN R. BROMBERG & LEWIS D. LOWENFELS, SECURITIES FRAUD AND COMMODITIES FRAUD § 2.5(6) (2d ed. 1993).

(c) To engage in any act, practice, or course of business which operates or would operate as a fraud or deceit upon any person,

in connection with the purchase or sale of any security.¹⁰

The text of Rule 10b-5 does not expressly provide a private right of action. This absence of positive language detailing a private right of action remains the root of pervasive uncertainty and ambiguity in Rule 10b-5 jurisprudence. Nevertheless, courts have long held that Rule 10b-5 implies a private right of action under which investors may seek a remedy for injury independent of any enforcement action undertaken by the SEC.¹¹

Despite recent decisions limiting the expansive reach of Rule 10b-5,¹² it remains the chief private remedy against fraud in the purchase and sale of securities. As a threshold matter, establishing federal jurisdiction is usually not a problem. A plaintiff need only demonstrate that the defendant conducted some aspect of the contested securities transaction using an instrumentality of interstate commerce. Interstate telephone calls and interstate use of the mails certainly qualify; notably, *intrastate* telephone calls or use of the mails qualify as well.¹³ Moreover, a defendant cannot assert a lack of jurisdictional means merely by arguing that the violative misrepresentation or omission was itself never transmitted using an instrumentality of interstate commerce; any memorandum or telephone call transmitted using an instrumentality of interstate commerce, preceding or following the alleged violation, provides sufficient jurisdictional

¹⁰ 17 C.F.R. § 240.10b-5 (1995).

¹¹ See *Fratt v. Robinson*, 203 F.2d 627 (9th Cir. 1953); *Fischman v. Raytheon Mfg. Co.*, 188 F.2d 783 (2d Cir. 1951); *Kardon v. National Gypsum Co.*, 69 F. Supp. 512 (E.D. Pa. 1946). The Supreme Court conclusively affirmed an implied cause of action under § 10(b) and Rule 10b-5 in *Herman & MacLean v. Huddleston*, 459 U.S. 375 (1983).

¹² See *Central Bank v. First Interstate Bank*, 114 S. Ct. 1439 (1994) (denying an implied private right of action against aiders and abettors of securities law violations); *Lampf, Pleva, Lipkind, Prupis & Petigrow v. Gilbertson*, 501 U.S. 350 (1991) (holding that an action must be commenced within one year after discovering the facts constituting the violation and within three years of the violation); *Santa Fe Indus., Inc. v. Green*, 430 U.S. 462 (1977) (holding that the alleged conduct must be "deceptive"); *Ernst & Ernst v. Hochfelder*, 425 U.S. 185 (1976) (holding that a defendant must act with scienter, not merely negligence).

¹³ See, e.g., *Loveridge v. Dreagoux*, 678 F.2d 870, 874 (10th Cir. 1982); *Dupuy v. Dupuy*, 511 F.2d 641, 642 (5th Cir. 1975), *cert. denied*, 434 U.S. 911 (1977); *Ingraffia v. Belle Meade Hosp. Inc.*, 319 F. Supp. 537 (E.D. La. 1970). But cf. *Dennis v. General Imaging, Inc.*, 918 F.2d 496, 500 (5th Cir. 1990) (noting, without holding, that an intrastate telephone call may not satisfy the jurisdictional "in commerce" requirement).

means for the entire transaction. Importantly, however, face-to-face conversations do not independently establish jurisdictional means.

The transactional scope of Rule 10b-5 is equally broad. Rule 10b-5 provides three sweeping and widely overlapping causes of action. Each cause of action requires a plaintiff to sustain the burden of proof for five substantive elements. First, clauses (a) and (c) of Rule 10b-5 require a showing of a fraud or deceit, while clause (b) requires a showing of a misrepresentation or omission of a material fact. Next, a plaintiff must demonstrate that the fraud, deceit, misrepresentation or omission was perpetrated (2) by any person (3) in connection with (4) the purchase or sale (5) of any security. Finally, the elements of common law fraud overlay the five substantive elements of Rule 10b-5. Thus, a plaintiff must also establish materiality, reliance, causation and damages.

A plaintiff relying on clause (b), for example, must prove the existence of a misrepresentation or omission of fact by the defendant and her reliance upon the defendant's misrepresentation or omission of fact when making her investment decision. "Reliance" requires that the misrepresentation constitute a substantial factor in the plaintiff's investment decision.¹⁴ The "purchase or sale" requirement likewise demands that the plaintiff actually purchased or sold the securities involving the alleged misrepresentation, omission, fraud or deceit.¹⁵ In addition, the plaintiff must prove that the misrepresentation or omission of fact was material, that the defendant intentionally or recklessly made the misrepresentation or omission of fact, and that the plaintiff suffered damages as a result of the defendant's conduct.¹⁶ "Materiality" requires that there be a substantial likelihood that a reasonable investor would consider the disclosure of the omitted fact as significant in making an investment decision.¹⁷ Causation is established by proving a sufficient causal connection between the injury

¹⁴ See *Ernst & Ernst*, 425 U.S. at 206; *Lipton v. Documentation, Inc.*, 734 F.2d 740, 742-43 (11th Cir. 1984), *cert. denied*, 469 U.S. 1132 (1985); *List v. Fashion Park, Inc.*, 340 F.2d 457, 463 (2d Cir.), *cert. denied sub nom.*, *List v. Lerner*, 382 U.S. 811 (1965).

¹⁵ See *Blue Chip Stamps v. Manor Drug Stores*, 421 U.S. 723, 754-55 (1975).

¹⁶ See *Ross v. Bank South, N.A.*, 837 F.2d 980, 991 (11th Cir. 1988), *reh'g granted*, 885 F.2d 723 (11th Cir. 1989) (en banc), *cert. denied*, 495 U.S. 905 (1990); *Huddleston v. Herman & MacLean*, 640 F.2d 534, 543 (5th Cir. 1981), *aff'd in part and rev'd in part*, 459 U.S. 375 (1983); *Dupuy v. Dupuy*, 551 F.2d 1005, 1014 (5th Cir.), *cert. denied*, 434 U.S. 911 (1977).

¹⁷ See *Basic Inc. v. Levinson*, 485 U.S. 224, 236 (1988); *TSC Indus., Inc. v. Northway, Inc.*, 426 U.S. 438 (1976); *Huddleston*, 640 F.2d at 534.

suffered and the defendant's wrongful conduct.¹⁸ Finally, the plaintiff must quantify the alleged damages.

B. Calculation of Damages

1. The Out-of-Pocket Measure

The out-of-pocket measure has become a popular method of calculating damages in Rule 10b-5 cases.¹⁹ The out-of-pocket measure "fixes recovery as the difference between the purchase price and the value of the security at the date of purchase less the difference between the sale price and the value of the security at the date of sale."²⁰ One simple explanation for calculating damages by the out-of-pocket measure is expressed in the opinion of the Ninth Circuit in the case of *Green v. Occidental Petroleum Corp.*²¹ Judge Sneed, in a concurring opinion, articulated that:

[I]t becomes necessary to establish, for the period between the date of the misrepresentations and the date of disclosure, data which when arranged on a chart will form, on the one hand, a "price line" and, on the other, a "value line." The price line will reflect, among other things, the effect of the corporate defendant's wrongful conduct. The establishment of these two lines will enable each class member purchaser who has not disposed of his stock prior to disclosure of the misrepresentations to compute his damages by simply subtracting the true value of his stock on the date of his purchase from the price he paid therefor.²²

Under the out-of-pocket procedure, as detailed by Judge Sneed, calculating the value line is tantamount to calculating damages.

This conclusion is readily evident when viewed in graphic detail. The procedure is depicted in Figure 1, where the horizontal axis represents time and the vertical axis represents the per unit price of the security. For the purpose of illustration, suppose that the security

¹⁸ See *Wilson v. Ruffa & Hanover, P.C.*, 844 F.2d 81, 85-87 (2d Cir. 1988); *First Interstate Bank of Nev. v. Chapman & Cutler*, 837 F.2d 775, 779-80 (7th Cir. 1988).

¹⁹ See Bradford Cornell & R. Gregory Morgan, *Using Finance Theory to Measure Damages in Fraud on the Market Cases*, 37 UCLA L. REV. 883, 885 (1990); see also *Wool v. Tandem Computers, Inc.*, 818 F.2d 1433, 1436-37 (9th Cir. 1987); *Harris v. Union Elec. Co.*, 787 F.2d 355, 367 (8th Cir. 1986); *Huddleston*, 640 F.2d at 555-56; *Blackie v. Barrack*, 524 F.2d 891, 909 (9th Cir. 1975); *In re LTV Sec. Litig.*, 88 F.R.D. 134, 148, 148-49 (N.D. Tex. 1980); Arnold S. Jacobs, *The Measure of Damages in Rule 10b-5 Cases*, 65 GEO. L.J. 1093, 1099-1102 (1977).

²⁰ Cornell & Morgan, *supra* note 19, at 885.

²¹ 541 F.2d 1335 (9th Cir. 1976).

²² *Id.* at 1344 (Sneed, J., concurring).

prices represented in Figure 1 are that of the common stock of XYZ, a publicly held corporation whose shares are actively traded on a national exchange. Suppose further that XYZ's CEO withholds information from the public regarding XYZ's research and development that would be extremely depressing to the value of the firm, as represented by the price of its common stock, if the information became public. Before the commencement of the misrepresentation by the CEO, the price line and the value line of the XYZ common stock are coincident. However, once the misrepresentation begins, the value line, which represents the price the stock would assume absent the misrepresentation, declines significantly. The price line, which represents the actual market price of the security, remains above the value line for the duration of the time the misrepresentation is conducted. Once the misrepresentation is disclosed, the price line and the value line quickly return to coincidence. This result follows from the assumption of the ECMH that the price of a risky asset in an efficient market reflects information quickly and without bias.

For an investor who purchased shares of XYZ after commencement of the misrepresentation and did not sell until after the corrective disclosure, the measure of damages is the difference between the price line—which represents how much the investor actually paid for a share of XYZ—and the value line—which represents how much the investor should have paid for a share of XYZ in the absence of the misrepresentation. For an investor who purchased shares after com-

mencement of the misrepresentation, but who sold those shares before the corrective disclosure, the measure of damages is the difference between the price line and the value line at the time of purchase and the price line and the value line at the time of sale. Note that, where the difference between the price and value lines at the time of sale equals or exceeds the difference at the time of purchase, the investor has not suffered a loss and therefore may not pursue a claim for recovery of damages.

2. Determination of the Value Line

Determining the value line requires two steps: first, the corrective disclosure date and the resulting disclosure price must be ascertained; and, second, an appropriate model of asset pricing must be used to calculate the prices comprising the value line for the period commencing with the fraudulent conduct and ending with the corrective disclosure.

a. *Disclosure Date and Disclosure Price*

Accurate identification of the disclosure date is necessary because the value line is determined by moving backward in time from the corrective disclosure date to the date of the fraudulent conduct. Determining the disclosure date, however, can prove problematic. In most instances, the fraudulent conduct consists of a series of acts or omissions that are later remedied by yet another series of acts or admissions.²³

The disclosure price is "the price at which the security would have traded if the omitted and misrepresented information—and *only* that information—were accurately disclosed at the start of the class period."²⁴ Two steps are required to determine the disclosure price: (1) a determination of the precise nature of the information that is the subject of the fraudulent conduct; and (2) an estimation of the value of that information—i.e., estimating the effect of that information on the price of the security had the information been disclosed. Step two is generally viewed as the more problematic step because "[i]n order to calculate the equivalent disclosure price, one must estimate

²³ See *Backman v. Polaroid Corp.*, 910 F.2d 10, 17-18 (1st Cir. 1990). The parties disputed the disclosure date, the type of disclosure that should have been made, and the date on which the undisclosed information should have been disclosed in the first instance.

²⁴ Cornell & Morgan, *supra* note 19, at 894.

how disclosure of the omitted or misrepresented information would affect investor beliefs regarding the magnitude of future cash payouts on the security and the likelihood of receiving those payments.”²⁵ Unfortunately, capital market theory does not provide a quantitative framework for the purpose of calculating investor assessments of information. It only offers a framework for determining the price of a given security given the presumption of investor assessment of information. Therefore, determining disclosure price is necessarily fact-specific and obtainable only by inference.

b. *Calculating the Value Line*

There are two methods for calculating the value line that rely on capital market theory: (1) the comparable index approach; and (2) the event study approach. The comparable index approach relies on asset-pricing models to calculate estimates of the returns the security would have generated in the absence of the fraudulent conduct. The event study approach treats corrective disclosures “as events and substitutes the predicted return on the event days”²⁶ for those of the days during the fraud. The event study approach therefore assumes that the price and value lines will move in perfect correlation with each other, albeit at different levels, during the period of fraudulent activity; in contrast, the comparable index approach assumes that the price and value lines will only move in perfect correlation with each other by coincidence. These approaches are illustrated in Figure 2.

Note that Figure 2 illustrates that both approaches obtain the same price for the security on the date of the corrective disclosure. Prior to the corrective disclosure date, however, the approaches typically obtain different value lines. This difference results from the fact that the comparable index approach assumes that all price movements that cannot be explained by reference to the security’s covariation with a general market factor are due to the fraudulent activity. By contrast, the event study approach assumes that the change of price on the disclosure date best describes the influence of the fraudulent activity for all points in time during the period of fraud, irrespective of a general market factor.

²⁵ *Id.* at 895.

²⁶ *Id.* at 897.

The shortcomings of each approach have been widely recognized in the literature.²⁷ Those of the comparable index approach have been summarized as follows:

The trouble with the comparable index approach . . . is that it attributes any decline in the security price that is not due to movements in the market or the industry to disclosure of the fraud. If the disclosure of a fraud is associated with the release of other company-specific bad news, the comparable index approach will overestimate the true damages.²⁸

Not surprisingly, the trouble with the event study is the opposite:

[B]y substituting predicted returns for actual returns only on disclosure dates[,] . . . the event study procedure will be biased if there is leakage of information. . . . This prior information leak means that the difference between the predicted return and the actual return . . . does not properly measure the economic impact of the disclosure. As a result, a value line which substitutes predicted returns for actual returns only on disclosure days will understate damages.²⁹

²⁷ See *id.* at 903-11; Frank H. Easterbrook & Daniel R. Fischel, *Optimal Damages in Securities Cases*, 52 U. CHI. L. REV. 611, 626-30 (1985); Daniel R. Fischel, *Use of Modern Finance Theory in Securities Fraud Cases Involving Actively Traded Securities*, 38 BUS. LAW. 1, 17 (1982); Jared Tobin Finkelstein, Note, *Rule 10b-5 Damage Computation: Application of Financial Theory to Determine Net Economic Loss*, 51 FORDHAM L. REV. 838, 854-58 (1983) [hereinafter Note, *Rule 10b-5 Damage Computation*]; Philip J. Leas, Note, *The Measure of Damages in Rule 10b-5 Cases Involving Actively Traded Securities*, 26 STAN. L. REV. 371, 393-95 (1974).

²⁸ Cornell & Morgan, *supra* note 19, at 903.

²⁹ *Id.* at 903-05.

The empirical derivatives of the CAPM, discussed *infra*, are commonly used to calculate the value line under both the comparable index and event study approaches. A more detailed quantitative example of the calculation of the value line is offered later in the context of the discussion regarding those empirical derivatives.

II. THE MERGER OF JUDICIAL DOCTRINE AND FINANCIAL THEORY

A. *The Theory of Fraud-on-the-Market*

The judicial "hook" into capital market theory for the purpose of satisfying the reliance requirement in Rule 10b-5 cases, without requiring a plaintiff to bear the burden of positive proof of reliance on misstatements or omissions of fact, is known as the "fraud-on-the-market" theory. The fraud-on-the-market theory considers the effect of misstatements or omissions of fact on a security's price in an efficient market, rather than on an investor's specific cultivation of information material to making an investment decision. Thus, the fraud-on-the-market theory presumes that in an efficient market, investors rely on the market price of a security to reflect all material information about the security.³⁰ The assumption underlying the fraud-on-the-market theory is that "[t]he market is acting as the unpaid agent of the investor, informing him that given all the information available to it, the value of the stock is worth the market price."³¹

The Supreme Court recognized the fraud-on-the-market theory in *Basic Inc. v. Levinson*,³² thereby allowing a class action brought under Rule 10b-5 to proceed without positive proof that individual members of the class directly relied on the misrepresentation or omission of fact by the defendants in making their investment decisions. Basic Inc. was a manufacturing firm whose equity securities were traded on the New York Stock Exchange. Basic's officers entered into merger

³⁰ For an analysis of the Supreme Court's decision to observe the fraud-on-the-market theory, see generally Jonathan R. Macey & Geoffrey P. Miller, *Good Finance, Bad Economics: An Analysis of the Fraud-on-the-Market Theory*, 42 STAN. L. REV. 1059 (1990); see also Jonathan R. Macey et al., *Lessons from Financial Economics: Materiality, Reliance, and Extending the Reach of Basic v. Levinson*, 77 VA. L. REV. 1017 (1991) (criticizing the distinction between efficient and inefficient markets underlying the fraud-on-the-market theory adopted in *Basic Inc. v. Levinson*).

³¹ *Basic Inc. v. Levinson*, 485 U.S. 224, 244 (1988) (quoting *In re LTV Sec. Litig.*, 88 F.R.D. 134, 143 (N.D. Tex. 1980)).

³² 485 U.S. 224 (1988).

negotiations with Combustion Engineering in September 1976.³³ Basic's officers, however, made three public statements subsequent to the initiation of the merger negotiations in which they denied the existence of merger discussions.³⁴ Basic's board of directors approved a tender offer by Combustion Engineering in December 1978.³⁵ Shareholders of Basic equity securities who sold shares during the period commencing with the first public denial of merger negotiations and the ultimate approval of the merger with Combustion Engineering brought suit under Rule 10b-5. The shareholders alleged that Basic's misleading statements caused the price of Basic's shares to be artificially depressed, resulting in their receiving less for their Basic shares than they would have received in the absence of the officers' misrepresentations.³⁶

As explained by the Supreme Court in *Basic*, the fraud-on-the-market theory establishes a rebuttable presumption that

[a]n investor who buys or sells stock at the price set by the market does so in reliance on the integrity of that price. Because most publicly available information is reflected in market price, an investor's reliance on any public material misrepresentations, therefore, may be presumed for purposes of Rule 10b-5 action.³⁷

The Court based its adoption of the fraud-on-the-market theory on its implicit assumption of the validity of the principles underlying the ECMH, considered at length below. Although the Court did not state its acceptance of the ECMH by name, the Court unmistakably stated its acceptance of the ECMH in substance when it expressed that

"[t]he fraud on the market theory is based on the hypothesis that, in an open and developed securities market, the price of a company's stock is determined by the available material information regarding the company and its business. . . . Misleading statements will therefore defraud purchasers of stock even if the purchasers do not directly rely on the misstatements. . . . The causal connection between the defendants' fraud and the plaintiffs' purchase of stock in such a case is no less significant than in a case of direct reliance on misrepresentations."³⁸

³³ See *id.* at 226-28.

³⁴ See *id.* at 227 & n.4.

³⁵ See *id.* at 228.

³⁶ See *id.*

³⁷ *Id.* at 247.

³⁸ *Id.* at 241-42 (alteration in original) (quoting *Peil v. Speiser*, 806 F.2d 1154, 1160-61 (3d Cir. 1986)).

Importantly, the Court declined to delineate how to prove or disprove that a market for a particular security is efficient.³⁹ In addition, the Court failed to address the implications for the measurement of damages in Rule 10b-5 suits relying on the fraud-on-the-market theory and, by implication, the ECMH.⁴⁰

B. *The Efficient Market Hypothesis*

As noted above, the Supreme Court in *Basic* impliedly accepted the ECMH when it stated that: "[I]n an open and developed securities market, the price of a company's stock is determined by the available material information regarding the company and its business."⁴¹ A more refined view of the ECMH holds that prices adjust quickly and without bias to publicly available information, or in the words of Jensen, "[a] market is efficient with respect to [a given] information set . . . if it is impossible to make economic profits⁴² by trading on the basis of [that] information set."⁴³

The ECMH has been parsed into three categories:⁴⁴ the "weak" form, the "semi-strong" form and the "strong" form. The weak form of the ECMH holds that the price of a security reflects all the information impounded in the security's past price series.⁴⁵ The semi-

³⁹ On the issue of proving or disproving market efficiency for a particular security, see Brad M. Barber et al., *The Fraud-on-the-Market Theory and the Indicators of Common Stocks' Efficiency*, 19 J. CORP. L. 285, 290 (1994) ("[V]olume of trade and number of analysts [following a stock] consistently discriminate between the efficient and inefficient stocks in our sample . . .").

⁴⁰ See *Basic*, 485 U.S. at 248 n.28.

⁴¹ *Id.* at 241-42 (quoting *Peil*, 806 F.2d at 1160).

⁴² "Profits" is used as a term of art meaning "risk adjusted returns net of all costs." *Id.* If an investor purchases risky asset *X* instead of risky asset *Y*, where assets *X* and *Y* have identical risk—i.e., variation of return relative to the mean return of the asset—both *ex ante* and *ex post*, and thereby receives a return on her investment net of expenses greater than she would have received had she invested her funds in *Y*, she has realized a profit.

⁴³ Michael C. Jensen, *Some Anomalous Evidence Regarding Market Efficiency*, 6 J. FIN. ECON. 95, 96 (1978).

⁴⁴ For a more detailed consideration of the three categories of the ECMH, see Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 25 J. FIN. 383, 388-413 (1970).

⁴⁵ The weak form of the ECMH is consistent with the statistical definition of a random walk. In the context of our discussion, a random walk is a time series such that the expected price of an asset in any subsequent period is equal to its price in the preceding period. In other words, at any particular point in time, the amount and direction of price change for a particular asset is random with respect to the information available at that particular point in time. See *id.* at 386-87.

strong form holds that a security's current price not only reflects all information impounded in the series of previous prices but all publicly available information that is material to the security.⁴⁶ The strong form holds that the price of a security is an unbiased reflection of both public and private information material to the security.⁴⁷ In sum, the ECMH "embraces two different kinds of claims: [1] that all relevant information will be available to the market and [2] that the market rapidly, if not instantaneously, digests all information as it becomes available."⁴⁸

Importantly, the ECMH is just that—a hypothesis. The validity of the ECMH is not susceptible to verification for a very simple reason:

[E]very test of [ECMH] also assumes some particular theory of what the "right" price for an asset is. . . . [A]sset pricing models establish the benchmark of "normal" returns in order to determine the efficiency of the market. Consequently, every empirical test of the efficient market hypothesis is a "joint test" of both the hypothesis and an asset pricing model.⁴⁹

Further, as Gilson and Black note, the ECMH is

an extreme null hypothesis that can't be strictly true. Prices can become and remain "right" only if investors, in a constant search for bargains, work at getting them that way. Investors will engage in this effort only if there is profit in it. The profit, though, must come from inefficiencies of one sort or another. Thus, there must be enough inefficiency to induce investors to search for and trade on mispricing—an equilibrium level of inefficiency. The interesting empirical question is how close public securities markets come to being . . . efficient, not whether they are perfectly efficient.⁵⁰

Even though empirical testing of the ECMH is plagued by the irreconcilable joint hypotheses problem described above, many financial economists have undertaken empirical testing of the ECMH by

⁴⁶ Note that the semi-strong form of the ECMH subsumes the weak form.

⁴⁷ The strong form of the ECMH is the most comprehensive of the three forms and represents an extreme notion of market efficiency. In fact, the strong form is not satisfied in the real world. If it were, insider trading would not be the profitable activity it has been demonstrated to be. See Fama, *supra* note 44, at 409-10.

⁴⁸ Jeffrey N. Gordon & Lewis A. Kornhauser, *Efficient Markets, Costly Information, and Securities Research*, 60 N.Y.U. L. REV. 761, 770-71 (1985).

⁴⁹ *Id.* at 772.

⁵⁰ GILSON & BLACK, *supra* note 5, at 139; see also Sanford J. Grossman & Joseph E. Stiglitz, *On the Impossibility of Informationally Efficient Markets*, 70 AM. ECON. REV. 393, 393 (1980) (proposing and describing "a model in which there is an equilibrium degree of disequilibrium: prices [partially] reflect the information of informed individuals").

using empirical derivations of theoretical asset-pricing models. Many of those studies have concluded that the ECMH cannot be rejected at a meaningful level of statistical significance, while others have determined that the ECMH cannot be accepted at a meaningful level of statistical significance.⁵¹ Regardless of the aggregate ambiguity of the studies' conclusions, it is regarded as incontrovertible that a significant body of empirical evidence strongly suggests that the capital markets are highly efficient processors of information.⁵² Therefore, any disagreement regarding market efficiency is necessarily limited to a debate over the degree of efficiency—i.e., whether the semi-strong or the weak form of the ECMH better describes the capital markets.⁵³

For our purposes, we assume that the semi-strong form of the ECMH is an accurate representation of the operation of the capital markets in the United States where the subjects of the transaction at issue are actively traded, publicly held securities. If we were to treat the validity of the ECMH as an open question, an examination of asset-pricing models as applied to the measurement of damages in Rule

⁵¹ The increasing weight of the evidence against the ECMH prompted the editor of the *Financial Analysts Journal* to comment in 1984:

Enter an abundance of idiosyncrasies—small firm effect, turn-of-the-year effect, low price-earnings ratio, junk bonds (stocks?), low-priced stocks, the Value Line phenomenon, weekend effects, performance of low beta portfolios, sector rotation, and information coefficients. Documented idiosyncratic market phenomena, like crocuses, herald a new season. The question is: How long can the [ECMH] continue, unrevised, against the burgeoning list of idiosyncratic phenomena?

Charles A. D'Ambrosio, *Toward an Idiosyncratic Marketplace*, FIN. ANALYSTS J., Mar.-Apr. 1984, at 9, 9.

⁵² Much of the empirical evidence in support of the ECMH was compiled prior to the market crash of October 1987. Since the crash, many of the financial economics community's more notable scholars have examined the implications of the crash to the ECMH, as the crash would appear to be a rather compelling empirical refutation of the ECMH. For a sampling of the literature, see Fischer Black, *An Equilibrium Model of the Crash*, in NBER MACROECONOMICS ANNUAL 1988, at 269 (Stanley Fischer ed., 1988); Eugene F. Fama, *Perspectives on October 1987, or, What Did We Learn from the Crash?*, in BLACK MONDAY AND THE FUTURE OF FINANCIAL MARKETS 71 (Robert W. Kamphuis, Jr. et al. eds., 1989); Kenneth R. French, *Crash-Testing the Efficient Market Hypothesis*, in NBER MACROECONOMICS ANNUAL 1988, *supra*, at 277; Richard Roll, *The International Crash of October 1987*, in BLACK MONDAY AND THE FUTURE OF FINANCIAL MARKETS, *supra*, at 35.

⁵³ Numerous articles of legal and economic scholarship that cast doubt on the validity of the ECMH have been published in recent years. See, e.g., Lawrence A. Cunningham, *From Random Walks to Chaotic Crashes: The Linear Genealogy of the Efficient Capital Market Hypothesis*, 62 GEO. WASH. L. REV. 546 (1994); Peter Fortune, *Stock Market Efficiency: An Autopsy?*, NEW ENG. ECON. REV., Mar.-Apr. 1991, at 17; Gordon & Kornhauser, *supra* note 48; Simon M. Keane, *The Efficient Market Hypothesis on Trial*, FIN. ANALYSTS J., Mar.-Apr. 1986, at 58.

10b-5 suits would be a significantly more difficult, if not futile, undertaking.⁵⁴ Given the assumption of efficient markets, attention may now turn to consideration of asset pricing, beginning with consideration of the relationship between risk and return.

C. Risk Compensation

The fundamental postulate of financial economics, and hence of the United States capital markets, is that the reward or return of a particular security is a positive function of the risk⁵⁵ or uncertainty of the reward associated with that security.⁵⁶ Therefore, "assets with the same risk should have the same expected rate of return. That is, the prices of assets in the capital markets should adjust until equivalent risk assets have identical expected returns."⁵⁷ The price of a risk-free asset reflects the value of time, whereas the price of a risky asset reflects both the value of time and the risk of loss. This relationship between risk and return reflects the notion that the majority of investors are risk averse.⁵⁸ Defining risk precisely, however, is one of the most difficult issues in determining 10b-5 damage.

⁵⁴ Note that if the *ex ante* forecast of an asset-pricing model matched the *ex post* result, two conclusions could be asserted: (1) the model is a valid description of the pricing of securities in the particular market and the market is efficient, or (2) the model is not a valid description of the pricing of securities in the particular market and/or the market is not efficient, but the *ex ante* forecast and the *ex post* result match by coincidence. Similarly, if the *ex ante* prediction of a model did not match the *ex post* result, two conclusions would obtain: (1) the model is valid but the market is not efficient, or (2) the model is not valid but the market is efficient.

⁵⁵ Risk in capital market theory is considered to be the chance of loss—i.e., either a negative return or a positive return that is less than a prescribed minimum. Where the return is certain over the holding period, there is no risk. One commonly used quantification of risk is the statistical measure known as variance (s^2), which in this context measures the degree of dispersion of returns about the expected or average return. The larger the variance, the greater the potential dispersion of expected returns, and therefore the greater the risk or uncertainty of returns.

$$s^2 = (1/n) \sum_{i=1}^n (\tau_{it} - R_t)^2, \text{ where } R_t = (1/n) \sum_{i=1}^n \tau_{it}$$

where τ_{it} is the return for asset i at time t , and n is the number of observations.

The standard deviation (s) of a variable is the square root of its variance. Standard deviation is an alternative means of measuring risk, $s = (s^2)^{1/2}$.

⁵⁶ For an early empirical study regarding risk compensation, see Fischer Black et al., *The Capital Asset Pricing Model: Some Empirical Tests*, in *STUDIES IN THE THEORY OF CAPITAL MARKETS* 79 (Michael C. Jensen ed., 1972).

⁵⁷ Franco Modigliani & Gerald A. Pogue, *An Introduction to Risk and Return: Concepts and Evidence*, *FIN. ANALYSTS J.*, May-June 1974, at 69, 69.

⁵⁸ A risk-averse investor is one who will "reject a fair gamble. For instance, given some initial wealth w_0 and a gamble to win or lose an amount x with equal probability,

In well-developed securities markets, such as those in the United States, it has been observed that the prices of securities and, therefore, their rates of return, move in conjunction with the aggregate market of securities. In other words, when the aggregate market advances (declines), the price and therefore the expected return of a security in that market is also likely to advance (decline). This covariance⁵⁹ of price fluctuation is due to a general market factor, i.e., an aggregate of economic forces that influence the earnings prospects of the majority of risky assets.

Given that a particular security is not likely to be so comprised such that its price fluctuations always reflect only the influences of the general market factor in a manner identical to that of the aggregate market, the risk of any particular security may be divided into two parts: systematic and unsystematic risk. Systematic risk for a security results from the covariance of returns between the security and aggregate market factors; unsystematic risk is the remaining variance of the stock unique to that stock, which is not related to market factors.

An obvious implication of these two types of risk is that an investor can reduce her risk by diversifying her securities holdings. This condition can be illustrated by the case where an investor adds securities to her portfolio until she has achieved a portfolio that is equivalent to the aggregate market. By taking such action, she eliminates from her portfolio all vestiges of unsystematic risk.⁶⁰ This type of diversification, where securities are added to the portfolio without con-

the investor will refuse the gamble." GORDON J. ALEXANDER & JACK CLARK FRANCIS, *PORTFOLIO ANALYSIS* 16 (Ezra Solomon ed., 3d ed. 1986). This example implies that a risk-averse investor requires a positive expectation of return as a prerequisite to her participation. In contrast, a risk-seeking investor will accept a fair gamble, and a risk-neutral investor will be indifferent to a fair gamble. The capital market theory is premised on the notion that, on average, investors are risk averse.

⁵⁹ "Covariance" (s_{im}) is a statistical term that refers to the comovement between random variables. A positive covariance indicates that the variables are more likely than not to change in the same direction; a negative covariance indicates that the variables are more likely than not to change in opposite directions.

$$s_{im} = (1/n) \sum_{t=1}^n (r_{it} - R_t)(r_{mt} - R_m),$$

where r_{it} and r_{mt} are the returns for assets i and m at time t , n is the number of observations, $R_t = (1/n) \sum_{i=1}^n r_{it}$, and $R_m = (1/n) \sum_{m=1}^n r_{mt}$.

⁶⁰ For empirical studies that examine the relationship between diversification and risk reduction, see Lawrence Fisher & James H. Lorie, *Some Studies of Variability of Returns on Investments in Common Stocks*, 43 J. BUS. 99 (1970); W.H. Wagner & S.C. Lau, *The Effect of Diversification on Risk*, FIN. ANALYSTS J., Nov.-Dec. 1971, at 48.

sideration of the covariances between securities, is referred to as simple diversification.

Markowitz diversification provides a contrast to simple diversification:

Markowitz diversification involves combining securities with less than perfect positive correlation in order to reduce risk in the portfolio without sacrificing any of the portfolio's return. In general, the lower the correlations (or, equivalently, covariances) of the assets in a portfolio, the less risky the portfolio will be. This is true regardless of how risky the assets of the portfolio are when analyzed in isolation.⁶¹

Like simple diversification, Markowitz diversification can significantly reduce risk, but it cannot entirely eliminate it—barring the unlikely case where an investor creates a portfolio of securities having perfectly negative covariance of returns. The concept of risk reduction through diversification is illustrated in Figure 3.

Importantly, under Markowitz diversification, the investor's expected return does not necessarily decrease as a result of the reduction of risk. This result follows from the observation that a portfolio's expected return is equal to the weighted average of the individual securities' expected returns, but a portfolio's variance is equal to the

⁶¹ ALEXANDER & FRANCIS, *supra* note 58, at 48.

weighted average of the individual securities' variances of return and covariances of return.⁶²

This concept of undiminished return in the event of diminished risk is illustrated by a simple example presented by Vasicek and McQuown:

Consider two different common stocks. Assume for simplicity that they both have the same expected rate of return, E . If a part of the wealth available for investment, call it x_1 , is allocated to one stock and the remaining part $x_2 = 1 - x_1$ is invested in another, the expected rate of return, E_p , on this two-issue portfolio is a weighted average of the two expected returns, or

$$E_p = x_1 E + x_2 E = E.$$

The expected return on the portfolio is thus equal to that of either stock, since we, of course, assumed them to be the same. The volatility of the two-issue portfolio, however, is less (as will be seen) than the volatility of either stock, if only we assume them to comove through time imperfectly.

According to a theorem in statistics, the variance V_p of the portfolio is computed as:

$$V_p = x_1^2 V_1 + x_2^2 V_2 + 2x_1 x_2 C_{12}.$$

In this equation, V_1 and V_2 are variances of the two stocks, respectively; C_{12} is the covariance of their returns, and x_1, x_2 are their weights in the portfolio. The covariance term is crucial to the effect of diversification. If the two issues fluctuate in price independently of each other, then the covariance term is zero, and it is always possible to choose the relative proportions x_1, x_2 in such a way that the risk of the portfolio is smaller than that of either stock taken separately. This is due to the effect of squaring numbers less than one. For instance, when the volatility of both stocks is the same, $V_1 = V_2$, and the covariance is zero, the risk of a portfolio of equal investment in each stock is

$$V_p = (.5)^2 V_1 + (.5)^2 V_2 = .5 V_1.$$

Thus, the portfolio has a variance equal to only one half of the variance of either stock. Since the expected return on the portfolio is not re-

⁶² The expected return, $(E)r_p$, of the portfolio is

$(E)r_p = \sum_{i=1}^n x_i (E)r_i$, where x_i is the proportion of the portfolio invested in asset i , and $(E)r_i$ is the expected return to asset i .

The expected variance (s^2) of the portfolio is $s^2 = \sum_{i=1}^n \sum_{k=1}^n s_{ik} x_i x_k$, where s_{ik} is the covariance of return between assets i and k .

See ALEXANDER & FRANCIS, *supra* note 58, at 43, 45.

duced (i.e., as shown above $E_p = E$), such a portfolio is clearly preferable to a single-issue portfolio for any investor who is averse to risk.

Typically, two stocks exhibit some positive comovement; therefore the covariance term cannot be realistically assumed to be zero. The reduction of risk in that case is not as large as if the two stocks were independent, but it always can be made smaller than the simple average risk of the two stocks. Hence, the amount of risk per unit of expected return can be decreased through diversification.⁶³

Given that unsystematic risk can be reduced by diversification—and, in theory, eliminated—an investor should expect to be rewarded only for her portfolio's exposure to systematic risk. The importance of the foregoing to the measurement of damages in Rule 10b-5 cases is relatively straightforward: An investor injured by fraudulent conduct with respect to the expected return for a security is injured only with respect to the unsystematic risk she suffers from the fraud. Therefore, the systematic risk compensation of the security during the period of the fraudulent activity must be separated from the total risk compensation, leaving only the unsystematic risk compensation. However, measurement of a security's level of systematic risk requires a valid description of the manner in which assets should be priced in an efficient capital market. The most prominent of such models is the Capital Asset-Pricing Model. As mentioned previously, the CAPM's empirical derivations have been applied—in error, as we will show—to the task of damages measurement in Rule 10b-5 cases with great regularity. Therefore, a brief examination of the CAPM is in order.

D. *The Capital Asset-Pricing Model*

The CAPM is a capital market theory which provides that, in equilibrium, a security will have an expected return that is a positive linear function of its covariance with the market portfolio.⁶⁴ The CAPM is characterized by the descriptive equation, $[E]r_i = r_f + b_i([E]r_m - r_f)$,

⁶³ Oldrich A. Vasicek & John A. McQuown, *The Efficient Market Model*, FIN. ANALYSTS J., Sept.-Oct. 1972, at 71, 76-77.

⁶⁴ "Market portfolio" as used here is a term of art. The market portfolio in the context of the CAPM specifically refers to

a portfolio containing all marketable assets in the proportions x_i , where $x_i =$ (total value of asset i) $\div$ (total value of all assets in the market). The reason all marketable assets must be in [the market portfolio] is simply that if an asset were not in [the market portfolio], no investor would own it and thus markets would not clear.

ALEXANDER & FRANCIS, *supra* note 58, at 109.

where $[E]r_i$ is the expected return on security i for the period, $[E]r_m$ is the expected return on the market portfolio for the period, r_f is the return on a risk-free⁶⁵ security for the period, and b_i is the sensitivity or risk coefficient of security i relative to the market portfolio—i.e., b_i is a measure of security i 's systematic risk. As detailed above, the risk coefficient of security i is defined algebraically as $b_i = s([E]r_i, [E]r_m) / s^2([E]r_m)$, and is equal to the slope of a straight line connecting the risk-free security with the market portfolio. "Thus, the covariance of [a security's] return with the market portfolio's return, not its variance of return, determines [a security's] expected return and is therefore the relevant measure of the [security's] risk."⁶⁶ In other words, the CAPM indicates that investors do not receive unsystematic risk compensation in equilibrium; they receive only the risk-free rate and a risk premium proportional to a security's level of systematic risk. In sum, the CAPM states that, given market conditions expressed in $[E]r_m$ and r_f , the expected return on a security is a linear function of the security's systematic risk. The greater the risk, the greater the expected return.

The assumptions underlying the CAPM are as restrictive as they are numerous. The following list represents the commonly accepted assumptions of the CAPM:

- (1) All investors have identical expectations about security rewards;
- (2) all investors have identical expectations about security risks;⁶⁷
- (3) investors experience identical net returns (taxes and investment expenses are identical);
- (4) there are no investment constraints (no limits on borrowing or lending, no short-selling restrictions, no upper bounds on holdings);⁶⁸

⁶⁵ Risk-free means that the nominal return for the period is known with certainty. It does not consider risks to the purchasing power of the principal from inflation.

⁶⁶ ALEXANDER & FRANCIS, *supra* note 58, at 113.

⁶⁷ Assumptions (1) and (2) express the notion that investors have "homogeneous expectations"—i.e., all investors "have the same one-period investment horizon and have the same perceptions regarding securities' *ex ante* expected returns, variances, and covariances." ALEXANDER & FRANCIS, *supra* note 58, at 107. For an argument about why heterogeneity of opinion is essential to the operation of capital markets, see Joram Mayshar, *On Divergence of Opinion and Imperfections in Capital Markets*, 73 AM. ECON. REV. 114 (1983).

⁶⁸ The assets are also infinitely divisible—i.e., partial rights to the assets may be exchanged.

(5) there is a risk-free asset, which is borrowed or lent at identical rates;⁶⁹

(6) all investors maximize mean/variance utility functions⁷⁰ over a common investment horizon and are risk-averse;

(7) investors experience risk only from the [market] portfolio (there are no risky assets or liabilities excluded from the problem);

(8) markets are perfect (each investor is a price-taker who does not believe he can influence price, there are no transaction costs and no costs of acquiring information).⁷¹

Note that these assumptions do not allow for diversity of investment objectives and holding periods. They also eliminate consideration of disparate information acquisition across investors. Note also that Rosenberg has observed that the CAPM assumptions necessarily imply the following:

(1) Each individual investor's portfolio satisfies the Markowitz condition;⁷²

(2) each investor's portfolio of risky assets has the same composition as all other investors';⁷³

(3) the market portfolio, which is the aggregate of all portfolios, therefore has this same composition;

(4) hence, the market portfolio is efficient⁷⁴ for all investors, the unique "mutual fund" of all risky assets that exactly suits the needs of all investors;

⁶⁹ More refined versions of the CAPM consider the implications of a higher rate of interest for borrowing, see Fischer Black, *Capital Market Equilibrium with Restricted Borrowing*, 45 J. BUS. 444 (1972); Black, *supra* note 56, at 98.

⁷⁰ For a discussion of utility functions and their role in portfolio theory, see ALEXANDER & FRANCIS, *supra* note 58, at 8-41.

⁷¹ Barr Rosenberg, *The Capital Asset Pricing Model and the Market Model*, J. PORTFOLIO MGMT., Winter 1981, at 5, 6.

⁷² The Markowitz condition is a consequence of Markowitz diversification and holds that there exists an infinite number of combinations or portfolios of risky assets for which their expected returns are superior to all other combinations having identical levels of expected risk or variance of returns. This set of portfolios is known as the efficient frontier. See HARRY M. MARKOWITZ, *PORTFOLIO SELECTION: EFFICIENT DIVERSIFICATION OF INVESTMENTS* (1959); Harry Markowitz, *Portfolio Selection*, 7 J. FIN. 77 (1952).

⁷³ This condition is known as the "separation theorem" which states that "the optimal investment decision is to buy what is known as the market portfolio This investment decision is, in turn, separate (meaning independent) from the decision about how to finance it—that is, whether to lend or borrow at the risk-free rate." ALEXANDER & FRANCIS, *supra* note 58, at 109.

⁷⁴ "An efficient portfolio has (1) greater expected return than any other portfolio in its risk class (that is, any other portfolio with the same variability of returns) and (2)

(5) since the market portfolio is efficient, any other portfolio of risky assets is inferior;⁷⁵

(6) investors price each security in the market [portfolio] so that its expected reward compensates for its contribution to risk in the market portfolio . . . ;

(7) hence, every portfolio . . . other than the market portfolio is inferior to the market portfolio because it has incremental diversifiable [unsystematic] risk.⁷⁶

Thus, in the world of the CAPM, investors compete with one another to assemble efficient portfolios.⁷⁷ As a consequence of this competition, "an asset's price reflects only the risk it contributes to a perfectly diversified portfolio. The market compensates investors, through greater expected return[s,] . . . only for that element of risk that cannot be eliminated by [Markowitz] diversification."⁷⁸

Professors Gordon and Kornhauser provide a succinct and accessible exposition of the implications of the CAPM:

CAPM implies that efficient portfolios consist of a combination of the market portfolio and the risk-free asset, in positive or negative amounts. Because optimal diversification eliminates nonsystematic risk, the best portfolio is the market portfolio, which by definition is subject only to systematic risk. An investor who wishes a portfolio less risky than the market should reduce her holdings of the market portfolio and acquire risk-free assets. Conversely, an investor seeking greater risk should leverage her holdings of the market portfolio by borrowing. . . .

The [security] market line represents expected returns at particular levels of risk from combinations of the most efficient portfolio of risky assets (presumably the market portfolio *M*) with the risky asset in positive (lending) or negative (borrowing) amounts. The gap between the [security] market line and the efficient portfolio (of risky assets) frontier represents the gains from using [the] so-called Separation Theorem in the construction of portfolios.

For those more accustomed to algebra, consider the following example: Assume that the risk-free return [r_f] is 8%, that the expected re-

less risk than any other portfolio with the same level of expected return." *Id.* at 52. The set of all efficient portfolios is known as the efficient frontier.

⁷⁵ This means inferior in the context of mean-variance efficiency. The returns to the inferior portfolio are less than should be expected given the variance of the returns of the portfolio.

⁷⁶ Rosenberg, *supra* note 71, at 6.

⁷⁷ Note that "[i]f the CAPM were strictly true, there would be no active management [of securities portfolios]. All investor expectations would be identical, and all investors would hold a single 'consensus portfolio.'" *Id.* at 7.

⁷⁸ Gordon & Kornhauser, *supra* note 48, at 778.

turn on a low beta (0.5) stock portfolio is 12%, and that the expected return on the market portfolio (beta = 1.0) is 20%. An investor seeking a risk level (or beta) of 0.5 can improve on the 12% expected return available from a portfolio solely of risky assets by putting half her funds in the market portfolio and half in risk-free assets (Treasury bills, e.g.). Her expected return from such a portfolio is 14%. $((.5 \times .08) + (.5 \times .20) = .14)$.⁷⁹

Figure 4 illustrates these concepts.

Figure 4 also illustrates the following more general conclusions outlined by Vasicek and McQuown:

1. In an efficient market, every investor should be expected to hold a combination of the riskless asset and the market portfolio. Such combinations of assets dominate any other alternatives in the sense that they are subject to less risk for the same level of expected return. The proportions to be invested in the risk[-free] asset and in the market portfolio depend solely on each investor's tradeoff of risk and expected return.
2. In an efficient market, the expected return on each [security] in excess of the risk-free rate is related only to its beta. A stock with a beta twice as high as another stock exhibits twice as high an expected excess return.⁸⁰

⁷⁹ *Id.* at 780 n.46 (citation omitted).

⁸⁰ Vasicek & McQuown, *supra* note 63, at 81.

The first conclusion describes how rational investors should behave in an efficient market; the second conclusion predicts how an efficient market would clear, given the assumptions of the model. The first conclusion is normative, the second positive. As such, the second conclusion may be subject to empirical study.

III. MOVING FROM THEORY TO APPLICATION

A. *The Market Model*

Although the CAPM describes how investors should be compensated for *ex ante* risk that they take in investment, it does not show the *ex post* relationship between securities returns and the market. That *ex post* relationship is described by the market model. That is, the market model is a return-generating model that attempts to describe how capital markets price assets or provide risk compensation *ex post*, whereas the CAPM only considers *ex ante* risk compensation. The market model holds that the return of a security is a linear function of both a market factor common to all securities and an independent factor unique to the particular security. Thus, the market model divides the return to a security into the two components discussed earlier: systematic risk compensation and unsystematic risk compensation.

The market model may be expressed algebraically as follows: $r_i = \alpha_i + \beta_i r_m + \varepsilon_i$, where β_i , beta, is a constant and is a measure of the security's systematic risk;⁸¹ r_i is the return to asset i ; r_m is the return to the market proxy; α_i , alpha, is a constant and a measure of the security's abnormal return;⁸² and ε_i is a random-error term for security i having a mean of zero. If the market model can be justified for use in testing the CAPM, then its use can also be justified for the purpose of measuring returns to a security in positive or negative excess of the risk compensation the security would have earned in the absence of fraudulent conduct. Therefore, the market model would represent a viable methodology for the measurement of damages in Rule 10b-5

⁸¹ The use of the beta coefficient, expressed as $s(i,m)/s_m^2$, as a measure of risk depends upon the empirical validity of the market model. Beta may be justified as a measure of risk by reference to the portfolio selection approach of Markowitz or to the equilibrium approach of Sharpe and Lintner. See Marshall E. Blume, *On the Assessment of Risk*, 26 J. FIN. 1, 1-2 (1971).

⁸² "Abnormal return" is the return remaining after the security's rate of return is adjusted for its systematic risk compensation. Recall that in the world of the CAPM, unsystematic risk is not compensated, therefore, abnormal returns are never observed.

cases. Alexander and Francis present the following use of the market model to test the CAPM:

The market model and the CAPM, despite being two independent and distinctly different models, have certain links to each other. The market model is a return-generating model and is based on the simple assumption that security returns conform to a multivariate normal distribution. . . . [I]t can be expressed algebraically as

$$r_i = \alpha_i + \beta_i r_m + \varepsilon_i \quad [1]^{83}$$

where $\beta_i = \text{cov}(r_i, r_m) / \sigma_m^2$. The CAPM is an equilibrium one-period model that explains expected returns and is based on a set of assumptions given earlier [in this paper]. Algebraically, it was shown to be equal to

$$[E]r_i = r_f + ([E]r_m - r_f)\beta_i \quad [2]$$

Given the expected return on the market r_m , the market model expresses a security's expected return as

$$[E]r_i = \alpha_i + \beta_i [E]r_m \quad [3]$$

In comparing equations [2] and [3], it can be seen that if the CAPM is valid, then $\alpha_i = \beta_i(1 - r_f)$. As will be shown next, the CAPM is typically converted from an ex ante model to an ex post or empirical model by merging it with a return-generating model.

By assuming that the multivariate normal distribution of returns is stationary over time, equations [1] and [2] can be modified by adding time subscripts. Furthermore, it can be noted from equation [3] that $\alpha_i = r_i - \beta_i r_m$. Substituting this expression for α_i into the time-subscripted version of equation [1] results in a form of the market model that can be viewed as a return-generating model,

$$r_{it} = [E]r_i + \beta_i(r_{mt} - [E]r_m) + \varepsilon_{it} \dots [4]$$

⁸³ Note that α_i and β_i are constants for the security i ; r_m is the return on an underlying factor as quantified through its influence on the market portfolio proxy; and ε_i is the random-error term on the security i . Further note that Alexander and Francis observe that because the market model:

is based on the assumption that the joint probability distribution between r_{it} and r_{mt} is stationary and bivariate normal [where r_{it} and r_{mt} denote the returns to security i and the market proxy m for holding period t]. As a result of this assumption, the error term $[\varepsilon_{it}]$ has the following properties:

1. The ε_{it} 's average value is zero
2. The variance of ε_{it} . . . is constant
3. The error terms are uncorrelated with r_{mt}

ALEXANDER & FRANCIS, *supra* note 58, at 75.

⁸⁴ Note that $[E]r_i$ and $[E]r_m$ denote the expected return of security i and the market portfolio, respectively.

Substituting the expression for $[E]\dot{r}_i$ [as shown in a time-subscripted version of equation [2]] into equation [4] results in

$$r_{it} = r_{ft} + \beta_i(r_{mt} - r_{ft}) + \varepsilon_{it} \quad [5]$$

Now, rearranging equation [5] and adding an intercept term, the following ex post version of the CAPM is created:

$$r_{it} - r_{ft} = a_i + \beta_i(r_{mt} - r_{ft}) + \varepsilon_{it} \quad [6]$$

Taking the expected value of equation [6] and rearranging terms indicates that the intercept term a_i is equal to

$$a_i = ([E]\dot{r}_i - [E]\dot{r}_f) - \beta_i([E]\dot{r}_m - [E]\dot{r}_f). \quad [7]$$

Now the term $\beta_i([E]\dot{r}_m - [E]\dot{r}_f)$ must be equal to $([E]\dot{r}_i - [E]\dot{r}_f)$ by the CAPM. Accordingly, the true value of a_i is zero and sample estimates of it should not be significantly different from zero if the CAPM is valid and stationary over time.

Finally, if equation [1] is applied to sample data for a given stock that has been generated by the CAPM, then the estimated values of α_i and β_i will be biased in opposite directions if r_{ft} and r_{mt} are correlated over time. However, empirical studies indicate that estimates of β_i derived from the market model and CAPM are quite similar in magnitude. Thus, the conceptual problem of using the market model to estimate β_i when the CAPM is assumed to be valid does not seem to create any serious empirical problems.⁸⁵

The constants in the market model equations (α_i and β_i) are typically determined by ordinary least squares regression ("OLS").⁸⁶ The OLS regression equation represents a straight line drawn through a set of points that depict the relationship between the return on the security and the market proxy such that the sum of the squares of the distances between the points and the line is minimized.

As argued above, "the true value of a_i [alpha, which measures a security's abnormal return] is zero and sample estimates of it [such as those determined by the market model] should not be significantly different from zero if the CAPM is valid and stationary over time."⁸⁷ The implication of this observation for measurement of damages in Rule 10b-5 cases is straightforward. By use of the market model, it may be possible to determine the return or risk compensation structure of security i for a given period relative to the return of the market portfolio for the same period. Recall that it has been shown

⁸⁵ ALEXANDER & FRANCIS, *supra* note 58, at 116-17 (footnotes omitted).

⁸⁶ A detailed discussion of regression analysis may be found in Franklin M. Fisher, *Multiple Regression in Legal Proceedings*, 80 COLUM. L. REV. 702 (1980).

⁸⁷ ALEXANDER & FRANCIS, *supra* note 58, at 117.

above that if the CAPM is valid, a security will not earn an abnormal return. Therefore, determining that a security under scrutiny in a Rule 10b-5 case has in fact experienced an abnormal return during the period of the fraudulent conduct is equivalent to measuring damages because in the world of CAPM, only systematic risk is compensated.

Application of the market model to the measurement of damages is relatively straightforward and seductively simple. The process is analogous to Security Market Line ("SML") analysis:

The SML [expresses] . . . the relationship between risk and return predicted by the mean-variance model, namely that risk, as measured by beta, and expected return are linearly related. SML analysis interprets deviations in expected return from the SML as abnormal returns above or below what is appropriate or warranted for the amount of risk taken on.⁸⁸

In the measurement of damages, the market model is used to calculate the expected return for the security given its level of risk. From the calculated expected return, the corresponding expected price of the security is determined. The difference between the price required to result in the expected return and the actual market price is the amount of damage an investor may have sustained per unit of the security purchased or sold. For illustrative purposes, one example is provided below. Note that this example is not illustrative of every possible type of injury that may result from a misrepresentation that influences the price of the entity's securities.

Assume that XYZ corporation intentionally overstated its prospective earnings for the forthcoming quarter, thereby artificially inflating the price of its common stock. Later, XYZ made a corrective disclosure to remedy the misrepresentation, and at such time, the price of XYZ common stock dropped 30.13%. The estimate of damages for a particular plaintiff would be calculated by the following process:

First, the market model regression equation for XYZ common must be calculated for a period prior to the misrepresentation.

Second, the observed return of the market portfolio, in this case the S&P 500 for the period of the fraudulent conduct—the period beginning with the plaintiff's purchase of the security and ending with the date of corrective disclosure—must be calculated.

⁸⁸ Philip H. Dybvig & Stephen A. Ross, *The Analytics of Performance Measurement Using a Security Market Line*, 40 J. FIN. 401, 401 (1985); see also *supra* fig.4 (SML analysis).

Third, the estimated return of XYZ common must be calculated for the same period as used to calculate the return on the market portfolio. This is accomplished by applying the previously calculated market return to the market model regression equation derived for XYZ common.

Fourth, the implied price of XYZ common for the date of the plaintiff's purchase must be calculated using the estimated return calculated in step three.

Finally, the difference between the actual price paid by the plaintiff and the implied price of XYZ common on the date of the purchase is the measure of damages per unit of the security, provided this difference is positive. Where the plaintiff sold her holdings prior to the date of corrective disclosure, her measure of damages is the difference between the damages as measured for the purchase date and the sale date, provided this difference is positive.

The following is a quantitative illustration⁸⁹ of the preceding process, where the market model regression equation for XYZ common relative to the S&P 500 Index is assumed to be $r_{xyz} = .0005 + 1.5r_{sp500}$, and the plaintiff purchased one share of XYZ common after the misrepresentation of expected earnings at the market price of \$29.25; on the same date, the S&P 500 level was 473.28. The corrective disclosure date price of XYZ common was \$20.00; the S&P 500 Index level was 446.77. The return for the S&P 500 Index for the period is -5.6%, $((446.77 - 473.28)/473.28 = -5.6\%)$. The estimated return for XYZ common over the same period is -8.35%: $(.0005 + 1.5(-.056) = -8.35\%)$. The implied price of XYZ common for the date of the plaintiff's purchase is \$21.82: $(\$20.00/(1 - .0835) = \$21.82)$. The amount of damages sustained by the plaintiff is \$7.43: $(\$29.25 - \$21.82)$. Calculation of the difference between the market price and the estimated price for each day of the period of fraudulent conduct results in the value line discussed above.

B. Ex Ante Versus Ex Post Analysis and the CAPM

In the preceding Sections, we distinguished between the market model and the CAPM. In particular, as we noted, the CAPM is used to analyze in advance the expected trade-off between risk and return for a given security or a given portfolio of securities. The CAPM itself,

⁸⁹ This example is based upon the market model calculation of damages example featured in Note, *Rule 10b-5 Damage Computation*, *supra* note 27, at 869.

therefore, is inherently based on expectations about the future. The market model, on the other hand, examines the *ex post* relationship between the market return and the return for a single security or a portfolio of securities. The market model, therefore, is intrinsically concerned with *ex post* analysis. In fact, an *ex post* analysis is necessary for determining damages in Rule 10b-5 cases because those damages measure the difference between what the return for the security would have been without the fraud and what the return actually was, taking into account overall economic changes that occurred during the period of the fraud. In the case of the market model, those economic changes are reduced to a single factor: the return on the market. Thus, damages in 10b-5 cases devolve into a simple question: What would the return to this stock have been, given the actual *ex post* observations on market returns, the long term historical relation between *ex post* market returns, and returns for the stock? Damages occur to the extent that actual realized *ex post* returns for the stock differ from what the stock returns would have been based solely on the realized values of the market return.

Importantly, we contend that the legal literature and commentators have almost universally misunderstood the difference between using the CAPM in an *ex post* and an *ex ante* sense in the Rule 10b-5 damage context. In particular, the legal community, to date, has failed to appreciate how this difference affects the way that empirical analysis is performed to determine whether the CAPM is useful from an investment perspective. A proper analysis of the CAPM for investment purposes compares actual *ex post* realized returns on particular stocks with the *ex ante* data for those stocks to determine whether, over some apt horizon, the average return for high beta stocks is greater than the average return for low beta stocks. Such investment analysis employs, by its nature, *ex ante* data, because investment decisions at any time were made using data available up to that time.

In contrast, any analysis in a Rule 10b-5 damages case is necessarily *ex post*. All data on actual market returns, as well as returns on the individual stock, are available. Therefore, we can ask the question: What would the return have been for the stock based on the actual realized returns on the market? This contrariety between damage assessment and investment decisionmaking has important implications for how we interpret the literature on tests of the CAPM. The vast bulk of the finance literature on tests of the CAPM⁹⁰ focuses only on

⁹⁰ See, e.g., Eugene F. Fama & Kenneth R. French, *The Cross-Section of Expected Stock*

the predictive ability of the CAPM, not on its *ex post* explanation of returns. Consequently, these articles on the predictive power of the CAPM are completely irrelevant for determining whether the CAPM is valuable to measure damages under Rule 10b-5.

In fact, the only criterion that should be used in determining which model is best suited for measuring damages under Rule 10b-5 is how well that model *ex post* can describe the historical relationship between the return of the stock of the firm subject to the Rule 10b-5 action and general conditions in the economy. It makes little sense to assume that the market model, which tries to explain all returns for individual firms based on the returns for the market as a whole, would offer the best explanation for describing how a stock's returns would have changed in the absence of fraud. Importantly, however, any model which explains the *ex post* return of stocks will be highly relevant for measuring Rule 10b-5 damages. Thus, the important empirical question is whether or not the market model is the best model for explaining *ex post* returns for stocks. If the market model is not the best model for assessing Rule 10b-5 damages, then another, more sophisticated model is required.

IV. MULTI-FACTOR DAMAGE ASSESSMENT MODELS

A. General Models

When we were analyzing how to measure returns using the stock market as a whole as the only controlling factor, we were very clear in distinguishing between the CAPM, which is based on *ex ante* analysis, and the market model, which is based on *ex post* analysis. One question remains, however: Is the movement in general stock returns the only relevant factor to control for in assessing damages? If there are other factors that help to explain the *ex post* realized returns on an individual stock in a 10b-5 case, then these factors should also be used to determine the portion of the stock return not explained by the fraudulent action. Indeed, we contend that a more sophisticated model will yield more credible results than a simple market model.

But how do we develop such a model? The model that we really need for damage assessment is an *ex post* model, and there has been relatively little work in finance examining what *ex post* models best ex-

Returns, 47 J. FIN. 427 (1992); Eugene Fama & James MacBeth, *Risk, Return, and Equilibrium: Empirical Tests*, 81 J. POL. ECON. 607 (1973); Ravi Jagannathan & Zhenyu Wang, *The Conditional CAPM and the Cross-Section of Expected Returns*, 51 J. FIN. 3 (1996).

plain realized stock returns. There has been, however, a significant amount of work in finance analyzing *ex ante* models that attempt to predict what returns will be based on utilizing more than just the single overall stock market index. These models are generally called multi-factor models and have often been justified by the arbitrage pricing theory ("APT"). As part of our justification for using multi-factor models, we turn now to a discussion of the APT and its application in modern finance. Note, however, that since the APT is really an *ex ante* model, we will propose another model, an *ex post* model, for use in damage assessments in a multi-factor world. As we will demonstrate, one can analogize the model we will use to the APT, just as the market model is analogized to the CAPM.

APT, like the CAPM, is a theory of asset pricing that describes the cross-section of returns of assets in equilibrium as a linear function of systematic risk. However, unlike the CAPM, which explains the cross-section of returns as a function of the "covariance between asset returns and an endogenous preference-based aggregate,"⁹¹ APT explains the cross-section of returns as a function of the "covariance between asset returns and factors in the return generating process."⁹² APT holds that "the expected return on any asset is directly related to that asset's sensitivity to unanticipated movements in major economic factors."⁹³

In both the APT and the CAPM, it is assumed that unsystematic risk can be eliminated by proper diversification. The APT further assumes that competitive forces in the market quickly and without bias eliminate arbitrage profit opportunities. The absence of arbitrage opportunities means that "investor[s] cannot earn a positive expected rate of return on any combination of assets . . . without making some net investment."⁹⁴ The absence of arbitrage profits follows from the APT's assumption that investors prefer more return to less for a given level of risk and prefer less risk to more for a given level of return. Investors acting in accordance with this assumption will therefore change their portfolio asset composition by assuming both long and short positions when given an opportunity to increase return without

⁹¹ Jay Shanken, *Multi-Beta CAPM or Equilibrium-APT?: A Reply*, 40 J. FIN. 1189, 1189 (1985) (footnote omitted).

⁹² *Id.*

⁹³ Richard Roll & Stephen A. Ross, *The Arbitrage Pricing Theory Approach to Strategic Portfolio Planning*, FIN. ANALYSTS J., May-June 1984, at 14, 18.

⁹⁴ Michael A. Berry et al., *Sorting Out Risks Using Known APT Factors*, FIN. ANALYSTS J., Mar.-Apr. 1988, at 29, 30.

increasing risk or decrease risk without decreasing return. In summary, the APT assumes a world in which "the relationship between return and risk will be determined by self-interested investors who will exploit opportunities to build portfolios of short and long positions, while making zero investment [as the proceeds from the short positions finance the long positions] but certain, positive returns."⁹⁵

The APT return-generating process is characterized by the following linear K -factor model:

$$\tilde{R}_{it} = E_i + \sum_{k=1}^K b_{ik} \tilde{\delta}_{kt} + \tilde{\epsilon}_{it}, \quad [1]$$

where R_{it} is:

the return on asset i between dates $t-1$ and t , E_i is the asset's expected return, δ_{kt} is the realization of the k th common factor (normalized to have a zero population mean), b_{ik} is the sensitivity of the return of asset i to the k th common factor (called the factor loading), and ϵ_{it} is the . . . [unsystematic] return on the i th asset, which is assumed to have zero mean and finite variance, and to be sufficiently independent across securities so that . . . [unsystematic] risk can be eliminated in large, well-diversified portfolios.⁹⁶

The APT further assumes that investors "agree on both the factor coefficients, $b_{i[k]}$ and the expected returns, E_i ."⁹⁷ The return-generating process described by equation [1] states that the return to an asset is equal to the asset's expected return (E_i)—which is the sum of the asset's returns to anticipated changes in the systematic risk factors—plus the asset's unanticipated return ($b_{ik}\delta_{kt}$)—which is the sum of the asset's returns to unanticipated changes in the systematic risk factors—plus the asset's unsystematic risk return (ϵ_{it})—which is the return to the asset's unsystematic risk, a return which is assumed to have a mean of zero.

The developer of the APT, Stephen A. Ross,⁹⁸ as well as others, demonstrated that the

absence of riskless arbitrage opportunities implies that expected returns [E_i] must satisfy (approximately):

⁹⁵ Dorothy H. Bower et al., *Arbitrage Pricing Theory and Utility Stock Returns*, 39 J. FIN. 1041, 1043 (1984).

⁹⁶ Bruce N. Lehmann & David M. Modest, *The Empirical Foundations of the Arbitrage Pricing Theory*, 21 J. FIN. ECON. 213, 215 (1988).

⁹⁷ Richard Roll & Stephen A. Ross, *An Empirical Investigation of the Arbitrage Pricing Theory*, 35 J. FIN. 1073, 1082 (1980).

⁹⁸ See Stephen A. Ross, *The Arbitrage Theory of Capital Asset Pricing*, 13 J. ECON. THEORY 341 (1976).

$$E_i \approx \lambda_0 + b_{i1}\lambda_1 + \dots + b_{ik}\lambda_k, [2]$$

as the number of assets satisfying the factor model [1] tends toward infinity where λ_0 is the intercept of the pricing relation and λ_k is the risk premium on the k th common factor, $k = 1, \dots, K$.⁹⁹

Roll points out that in equation [2]

[i]f there is a riskless asset with return, E_0 , then $b_{0j} = 0$ and $E_0 = \lambda_0$, hence . . . [equation [2] may be expressed as follows]

$$E_i - E_0 = \lambda_1 b_{i1} + \dots + \lambda_k b_{ik},$$

with the understanding that E_0 is the riskless rate of return if such an asset exists, and is the common return on all 'zero-beta' assets, i.e., assets with $b_{ij} = 0$, for all j , whether or not a riskless asset exists.¹⁰⁰

A simple example of the APT relationship assuming a single factor is presented by Roll and reproduced here as illustrated in Figure 5:

If there is a single factor, then the APT pricing relationship is a line in expected return, E_i , systematic risk, b_i , space:

$$E_i - E_0 = \lambda b_i.$$

Figure [5] can be used to illustrate our argument geometrically. Suppose, for example, that assets 1, 2, and 3 are presently held in positive amounts in some portfolio and that asset 2 is above the line connecting assets 1 and 3. Then a portfolio of 1 and 3 could be constructed with the same systematic risk as asset 2, but with a lower expected return. By selling assets 1 and 3 in the proportions they represent of the initial portfolio and buying more of asset 2 with the proceeds, a new position would be created with the same overall risk and a greater return. Such arbitrage opportunities will be unavailable *only* when assets lie along a line. Notice that the intercept on the expected return axis would be E_0 when no arbitrage opportunities are present.¹⁰¹

⁹⁹ Lehmann & Modest, *supra* note 96, at 215.

¹⁰⁰ Roll & Ross, *supra* note 97, at 1078-79.

¹⁰¹ *Id.* at 1079.

Roll further notes that

[t]he pricing relationship $[E_i = \lambda_0 + \lambda_1 b_{i1} + \dots + \lambda_k b_{ik}]$ is the central conclusion of the APT . . . , but it is natural to ask what interpretation can be given to the $\lambda_{[kj]}$ factor risk premia. By forming portfolios with unit systematic risk on each factor and no risk on other factors, each $\lambda_{[kj]}$ can be interpreted as $\lambda_{[kj]} = E^{[kj]} - E_0$, the excess return or market risk premium on portfolios with only systematic factor k risk. Then $[E_i = \lambda_0 + \lambda_1 b_{i1} + \dots + \lambda_k b_{ik}]$ can be rewritten as,

$$E_i - E_0 = (E^1 - E_0)b_{i1} + \dots + (E^k - E_0)b_{ik}.^{102}$$

The assumptions of the APT, like those of the CAPM, tend to differ as the theoretical derivation of the model under investigation tends to vary from the original proponent's explication. As a general matter, it is acknowledged that the assumptions of the APT are less restrictive than those of the CAPM. In addition to the assumptions detailed above, the following assumptions or a subset of the following assumptions are common to theoretical derivations of the APT:

1. Each asset has small idiosyncratic [unsystematic] variance, i.e., $\text{var}(\epsilon_i)$ is small.
2. Each asset has small supply in the economy (at least in the limit).
3. There is a portfolio which, up to a constant, mimics factor j (at least approximately, perhaps in the limit).

¹⁰² *Id.*

4. Some agent holds a well-diversified portfolio that does not contain any idiosyncratic risk.
5. There is no arbitrage (directly or in some asymptotic sense).
6. There is Pareto efficiency and/or aggregation. . . .
-
8. All assets are in positive supply.¹⁰³

B. Identification of Risk Factors

If we want to identify the multiple factors that we might use in a multi-factor damage assessment model under Rule 10b-5 for *ex post* analysis of stock returns, one place to start is the empirical research done on *ex ante* multi-factor models of stock returns, namely the empirical versions of the APT. The factors that are significant in multi-factor *ex ante* models of stock returns may potentially also be significant in *ex post* models as well.

Empirical research¹⁰⁴ indicates that unanticipated changes in the following four economic factors are particularly relevant to explaining *ex ante* stock returns using the APT:

- (1) inflation;
- (2) industrial production;
- (3) risk premiums, as measured by the rate differential between low grade and high grade bonds; and
- (4) the slope of the term structure of interest rates.¹⁰⁵

The changes in the systematic factors that influence asset returns must necessarily be unanticipated. In an efficient market, anticipated changes have already been impounded into the expected return (E_i) of an asset.

Roll and Ross note that it is not surprising that the four economic factors detailed above have been found to be important determinants of asset returns:

¹⁰³ Philip H. Dybvig & Stephen A. Ross, *Yes, the APT Is Testable*, 40 J. FIN. 1173, 1175 (1985).

¹⁰⁴ Empirical research with equities is described in the following works: Stephen J. Brown & Mark I. Weinstein, *A New Approach to Testing Asset Pricing Models: The Bilinear Paradigm*, 38 J. FIN. 711 (1983); Lawrence Kryzanowski & Minh Chan To, *General Factor Models and the Structure of Security Returns*, 18 J. FIN. & QUANTITATIVE ANALYSIS 31 (1983); Marc R. Reinganum, *The Arbitrage Pricing Theory: Some Empirical Results*, 36 J. FIN. 313 (1981); Roll & Ross, *supra* note 97.

¹⁰⁵ Roll & Ross, *supra* note 93, at 19.

They appear in the traditional discounted cash flow (DCF) valuation formula.¹⁰⁶ Two of them—[unanticipated] changes in industrial production and unanticipated inflation—are related to the numerator in the DCF formula, i.e., to the expected cash flows themselves. Expected industrial production is a proxy for the real value of future cash flows. Inflation enters because assets are not neutral; their nominal cash flow growth rates do not always match expected inflation rates.

The other two variables would seem intuitively to be more related to the denominator in the DCF formula—i.e., to the risk-adjusted discount rate. The risk premium measure is an amalgam of investor attitudes toward risk-bearing and perceptions about the general level of uncertainty. The term structure of interest rates enters because most assets have multiple year cash flows and, for reasons relating to risk and time preferences, the discount rate that applies to distant [cash] flows is not the same as the rate that applies to [cash] flows in the near future.¹⁰⁷

Regardless of the rationale expressed by Roll and Ross, it is clear that determining the entire set of systematic risk factors having pervasive influence over the returns to assets is inherently speculative and ultimately cannot be conclusive. In fact, it has become fairly common to use the returns to a market index such as the S&P 500 Index as an additional systematic risk factor in an attempt to explain residual risk. The rationale for this practice is the belief that "[a]ny missing factor is embodied in this residual market factor in exactly the same manner that all factors are embodied in the market return for the market model or the CAPM."¹⁰⁸ However, the difficulty of not knowing precisely the identity of the systematic risk factors does not prevent the empirical application of the APT. Indeed, the ambiguity present in the APT seems slight when compared to the task of identifying the mean-variance efficient market portfolio prescribed by the CAPM.

C. *Application of the APT to the Measurement of Damages*

Measurement of damages in a Rule 10b-5 case by means of a multi-factor model is analogous to the procedure detailed in the Section regarding the market model. First, regression of the security's returns against the identified systematic risk factors for the period

¹⁰⁶ The concept of discounted cash flow may be expressed as follows:

$$P_t = E_t \sum_{k=0}^{\infty} [D(t+k)/(1+d)^{k+1}],$$

where P_t is the price of the asset at time t , E_t expresses expectations at time t , $D(t+k)$ is the cash flow available at time $t+k$, and d is the discount rate.

¹⁰⁷ Roll & Ross, *supra* note 93, at 19.

¹⁰⁸ Berry et al., *supra* note 94, at 31.

preceding the misrepresentation is performed. Second, the values of the systematic risk factors during the period of the misrepresentation are determined. Third, the estimated return of the security is calculated for the period of the misrepresentation. This estimate is derived by plugging into the multiple regression equation, which now expresses the security's risk coefficients for each systematic risk factor, the values of the unanticipated changes of the systematic risk factors. Fourth, the implied price or true value of the security for the date of plaintiff's transaction is calculated using the estimated return calculated in step three. Finally, the difference between the price of the security on the date of the plaintiff's transaction and the implied price represents the measure of damages per unit of the security, provided this difference is positive. As was the case in the example of calculating damages with the market model, the value line of a security as calculated with a multi-factor model is the set of implied prices calculated for each subperiod of the period of fraudulent conduct.

Any discussion proposing multi-factor models as an alternative to the market model for the purposes of measuring damages in a Rule 10b-5 case is incomplete without consideration of at least one empirical test that compares the multi-factor models and the single-factor market model. Therefore, the following Part summarizes the results of two different empirical studies and identifies insights regarding the implications for damages measurement.

V. MULTI-FACTOR AND SINGLE-FACTOR MODELS: A HORSE RACE

Throughout this Article, we have suggested that neither the CAPM nor the market model is adequate for measuring Rule 10b-5 damages. In this Part, we compare the explanatory power of single-factor models such as the CAPM and the market model with multi-factor models such as the APT. We summarize the results of two studies: the first compares the *ex ante* predictive power of the CAPM and the APT; the second compares the *ex post* explanatory power of the market model and a multi-factor model. While the former study is suggestive in favoring multi-factor models, it still focuses on finance applications of asset-pricing models, rather than on legal applications of asset-pricing models. This focus arises because the first study is concerned with *ex ante* prediction rather than *ex post* explanation. The second study directly addresses the question of which kind of model is more appropriate for measuring 10b-5 damages and shows that, at least in the sample considered, a multi-factor model is far superior to the market model.

In an important study, Bower, Bower and Logue estimated the relationship of risk and return for electric utilities and natural gas utilities by means of the CAPM and the APT.¹⁰⁹ They concluded that "APT does do better than CAPM in explaining and conditionally forecasting return variations through time and across assets," and that it "would be wise to give APT greater weight [than the CAPM] in decisions."¹¹⁰ Their conclusion rests on their findings that the APT not only better explains the variance of returns than does the CAPM, but that it also produces better out-of-sample forecasts of expected returns. This foundation is particularly noteworthy in connection with the measurement of damages in Rule 10b-5 cases. From our previous discussion, recall that the calculation of the expected return is the principle objective of measuring damages in a Rule 10b-5 case. From the estimated expected return, the true value or implied price of the security for the date of a plaintiff's transaction may be determined. The difference between the true value and a plaintiff's transaction price, if positive, is the per share measure of damages.

The evidence presented by Bower et al. was constructed using monthly returns for 942 stocks and four Treasury security portfolios from 1971 through 1979.¹¹¹ All of the stocks were listed on either the New York or the American Stock Exchanges and traded continuously from 1963 through 1980.¹¹² The stocks were divided into a total of thirty portfolios—eighteen portfolios contained thirty-one stocks and twelve contained thirty-two stocks.¹¹³ Bower et al. note that portfolios were used to reduce the "noisiness" of the data present in individual stocks return data.¹¹⁴ All companies within an industry group were placed together in separate portfolios, except in the case of very large industry groups.¹¹⁵ Most portfolios, however, comprised more than one industry because most of the industry groups comprised fewer than thirty-one companies. Of the 942 stocks selected, 127 were excluded from the initial estimates of the APT and CAPM.¹¹⁶ This hold-out sample was used to construct four portfolios that were then used to test the ability of each model to estimate expected returns. The

¹⁰⁹ See Bower et al., *supra* note 95, at 1053.

¹¹⁰ *Id.*

¹¹¹ See *id.* at 1045.

¹¹² See *id.*

¹¹³ See *id.*

¹¹⁴ *Id.*

¹¹⁵ See *id.*

¹¹⁶ See *id.*

holdout stocks consisted of seventy-seven electric utilities, twenty-three natural gas companies, six telecommunications utilities and twenty-one industrials.¹¹⁷

The market portfolio proxy for the CAPM was the CRSP value-weighted index—which included the 127 stocks withheld for the out-of-sample tests of predictive power.¹¹⁸ To calculate the APT factor weights, the techniques described by Oldfield and Rogalski and by Gultekin and Rogalski were used.¹¹⁹ The four factors observed were chosen on the basis of the earlier findings of Roll and Ross, as published in 1980.¹²⁰

The first item of evidence presented in favor of the APT concerned the ability of the models to explain the variance of returns for the portfolios constructed of the in-sample stocks. "The R^2 for the APT characteristic line was higher for each of the 30 portfolios and the average value for R^2 was 0.869 for the APT characteristic lines and 0.605 for the CAPM lines."¹²¹ Although these statistics support Bower et al.'s conclusion that the APT better describes the return-generating process than does the CAPM, the weight of these statistics is diminished by the fact that the APT factor weights were derived from the very returns that they purport to explain.

The second item of evidence presented in favor of the APT does not require the preceding qualification because the variance of returns explained by the APT played no part in estimating the factor weights.

[F]or the 127 stocks in our holdout sample . . . [t]he R^2 is higher for 102 of the 127 stocks using the APT factors. [In] addition, the explained variance with APT is significant at the 1 percent level for the 127-stock holdout sample and for six of seven industry groups when each industry is considered separately.¹²²

¹¹⁷ See *id.*

¹¹⁸ See *id.*

¹¹⁹ See *id.* (citing Gregory S. Oldfield & Richard J. Rogalski, *Treasury Bill Factors and Common Stock Returns*, 36 J. FIN. 337, 337-50 (1981)); N. Bulent Gultekin & Richard J. Rogalski, *Government Bond Returns, Measurement of Interest Rate Risk, and the Arbitrage Pricing Theory* (Amos Tuck School of Business Administration, Dartmouth College Working Paper, 1982).

¹²⁰ See Bower et al., *supra* note 95, at 1045 (citing Roll & Ross, *supra* note 97, at 1073-1103).

¹²¹ *Id.* at 1046.

¹²² *Id.*

To be sure, this evidence is compelling support for the proposition that the APT offers a better explanation of the return-generating process than does the CAPM.

The third item of evidence in support of the APT over the CAPM considered the ability of the models to estimate expected returns. Evidence from this aspect of the study is particularly relevant to the measurement of damages. Bower et al. estimated the expected returns for the 127-stock holdout sample for each month of the 1971-1979 period by applying the APT factor betas and the CAPM betas calculated for the holdout sample to the APT and CAPM equations developed exclusive of the 127-stock holdout sample. The power of the estimates was assessed using an approach suggested by Theil.¹²³

The Theil measure, U^2 , uses the sum of squared differences of each stock's average return for the 1971-1979 period, $[R_i]$, from its CAPM or APT forecast of return, $[r_i]$, and the sum of the squared differences of average return for each stock from the average return for all stocks, $[r_u]$ The smaller the ratio, the better is the model forecast relative to the naive forecast.¹²⁴

The Theil measures for the APT and the CAPM were 0.822 and 1.115, respectively.¹²⁵ These scores indicate that not only did the CAPM fail to better the APT, but it also failed to better the naive estimate of expected returns. The naive estimate of expected returns is expressed in the denominator of the Theil measure, which may be expressed as follows:¹²⁶

$$U^2 = \frac{\sum_{i=1}^n (R_i - r_i)^2}{\sum_{i=1}^n (R_i - r_u)^2}$$

The superiority of APT over CAPM was evident even though the CRSP value-weighted index, which was used as the market portfolio proxy for the CAPM, included the 127 stocks comprising the holdout sample. When Bower et al. included the 127 holdout stocks in the development of the APT model, the ability of the APT to estimate expected returns improved significantly. The R^2 s increased and the unexplained variance of returns decreased for the APT estimates.¹²⁷ Similar improvement was reflected by the Theil measure. The Theil

¹²³ See HENRY THEIL, APPLIED ECONOMIC FORECASTING (1966).

¹²⁴ See Bower et al., *supra* note 95, at 1049-50.

¹²⁵ See *id.* at 1050.

¹²⁶ See *id.*

¹²⁷ See *id.* at 1051.

measure for the APT estimates of expected return declined from 0.822 to 0.505. Bower et al. note that

[t]he forecast improvement and the distribution of that improvement across industry groups are consistent with a conclusion that entirely omitting a class of companies, such as utilities, from the [APT] factor estimation process will provide scores that fail to reflect factors to which [that class of companies] may be particularly responsive. This is a bias against APT in our holdout tests that makes the holdout findings presented . . . even more impressive.¹²⁸

Bower et al. conducted parallel holdout analysis of weekly returns, and found the results to be substantially similar to that produced from the monthly data.¹²⁹ Additional research using daily returns without a holdout sample demonstrated that the four-factor APT explained more of the variance of returns than did the CAPM.¹³⁰ Unfortunately, Bower et al. presented the particulars of the weekly and the daily return studies in anecdotal form only.¹³¹

Notwithstanding the significance of the Bower et al. study, it suffers from a serious limitation. The authors erroneously assume that the factors they estimate for firms that are not in the utilities or telecommunications industries can be used to estimate what the returns would have been in the holdout sample. They provide no basis for this assumption. To the extent there may be industry-specific differences in the factors themselves, there is absolutely no basis for the conclusions drawn in the Bower et al. study. While we recognize this problem, we include the Bower et al. results because it is the most important existing study that actually tried to implement the APT model.

A more substantive problem with the Bower et al. study, perhaps, is that it is really an *ex ante* rather than an *ex post* analysis and it is premised on unobservable factors based on regression residuals, rather than observable factors such as inflation or interest rates. The only multi-factor models that will work well for assessing damages under Rule 10b-5 are *ex post* models that explain asset returns over a specific time period based on observable economic factors, such as the difference between long and short term interest rates or the difference between interest rates on high quality and low quality bonds, in addition to the overall stock market's return.

¹²⁸ *Id.*

¹²⁹ *See id.* at 1052.

¹³⁰ *See id.*

¹³¹ *See id.*

Exactly this kind of study was recently undertaken by the authors of this Article.¹³² In that study, the authors utilized a sample of firms in the electric utility industry and examined the monthly returns for those stocks over the period from 1964–1989. As in the Bower et al. study, and almost all other asset-pricing studies, these stocks were aggregated into different portfolios—in this case, ten portfolios in total. The authors then compared the explanatory power of the market model with the explanatory power of a multi-factor model which included two factors in addition to the market index. The first factor, the difference between the yields on long term corporate bonds and long term government bonds, captures the risk premium for corporate bonds. This factor should become larger as economic conditions worsen because investors in corporate bonds would require more of a risk premium to compensate them for possible losses due to bankruptcy. The second additional factor in the Adams et al. multi-factor model was the difference between the yields on long term government bonds and Treasury Bills, which reflects the steepness of the term structure of interest rates.¹³³ The slope of the term structure of interest rates is extremely important for financial markets and stock valuation because an upward-sloping yield curve implicitly predicts that short term interest rates will be significantly higher in the future than they are today.¹³⁴ Adams et al. found that for the ten utility stock portfolios under consideration, the multi-factor model explained almost 41% more of the total stock returns across the portfolios over the sample period than did the market model.¹³⁵ The Appendix to this Article shows that in each of the ten portfolios of utility stocks, both the risk variable and the term structure variable were statistically significant at the one percent level. These results suggest quite clearly that the market model is significantly inferior to a multi-factor model in assessing 10b-5 damages.

The implication for measurement of damages in Rule 10b-5 cases seems clear: Using the multi-factor model and stock returns for companies in the manipulated security's industry class during the period

¹³² Edward S. Adams, John W. Krainer & David E. Runkle, *Analyzing Stocks in the Electric Utility Industry* (1996) (unpublished manuscript, on file with authors).

¹³³ The term structure of interest rates is simply the relationship between the time to maturity for government bonds and their yield to maturity.

¹³⁴ See, e.g., FRANK J. FABOZZI, *BOND MARKETS, ANALYSIS AND STRATEGIES* 98-100 (3d ed. 1996).

¹³⁵ Adams et al. also examined other factors that had been included in multi-factor models, such as inflation and industrial production used in Roll & Ross, *supra* note 93, at 19.

of the manipulation can provide additional information that not only explains the variance of returns at a statistically significant level, but more importantly, produces better estimates of expected return for the period of manipulation than can be calculated by the market model.

While the results of the above-mentioned study are significant, it is more useful perhaps to demonstrate in concrete terms the superiority of multi-factor models over single-factor models in computing damages in securities fraud cases. Accordingly, consider the following illustration of the superiority of the multi-factor model in the context of a securities fraud class action suit brought against the Fleet/Norstar Financial Group.¹³⁶

In the Fleet/Norstar Financial Group case, the plaintiffs brought a class action suit against Fleet alleging that in 1989 and 1990 the defendants materially misrepresented the financial condition of Fleet. The plaintiffs alleged that this misrepresentation inflated the market price of the corporation's stock. To verify the superiority of our technique over conventional single-factor methodologies, we estimated the predicted excess returns to Fleet stock during the class period using each of the two respective models and compared the predicted returns from those models. In each case, we used monthly data from the five years prior to the beginning of the class period to estimate the parameters of the model for the purpose of determining the cumulative excess return above the appropriate Treasury Bill returns. Since the class period began in March of 1989, our estimation period began in March of 1984 and extended through February of 1989.

Prior to proceeding with the analysis, there were a number of compelling intuitive reasons to believe that the multi-factor model discussed in this Article would show substantially different estimates of damages than a single-factor market model. Whereas the market model seeks to explain all returns to Fleet stock as a function of overall stock price movements, for banks, both the term structure of interest rates and the spread between corporate bonds and conventional government bonds may have a significant effect on return to the stock. When long term interest rates are substantially higher than short term interest rates, bank profitability is likely to increase because the relative cost of a given bank's short term deposits will have declined compared to its returns on lending, at least for fixed rate

¹³⁶ *In re Fleet/Norstar Sec. Litig.*, 935 F. Supp. 99 (D.R.I. 1996).

loans. This increase in profitability should increase the value of bank stocks even if the overall market does not, in fact, move upward. Correspondingly, if the interest rate spread between long term government bonds and long term corporate bonds increases, that increase may indicate that banks are charging higher interest rates on loans, thereby increasing bank profits and bank stock returns as well.

As one might expect, the models show substantially different estimates for what the return to Fleet stock would have been without any fraud. The market model predicts that from the beginning of March of 1989 through the end of March of 1990, the return on Fleet stock would have been 16.64%. However, results from our three-factor model predict that the returns over that period would have been only 12.88%, given the actual stock market return term structure premium and risk premium between March of 1989 and March of 1990.¹³⁷ The two models' predictions differ by 3.76% over this period. Since the single-factor model estimates a higher return for Fleet stock than the multi-factor model, the estimate of damages from the single-factor model would be substantially larger than the estimate of damages from the multi-factor model (because the difference between the model's predicted return and the actual return of -19.42% during this period would be larger).¹³⁸ Since the value of Fleet's common stock at the end of February of 1989 was slightly over \$2.6 billion, this 3.76% difference in the two models' return represents an almost \$100 million difference in the value of the firm by the end of the class period.¹³⁹

This example illustrates the clear superiority of the multi-factor model compared to a single-factor market model in estimating securities returns, and hence damages, in securities fraud cases. Importantly, two caveats are necessary in assessing the new model. First, although the damage estimate in this case would be lower using the

¹³⁷ This particular model is estimated with monthly data and our results, just for illustration, are shown for monthly data. The actual class period for this suit was March 14, 1989 through April 3, 1990. See *id.* at 102. Obviously, to determine returns for this actual class period, a different model from the one used here would be necessary. This model is for illustrative purposes only.

¹³⁸ Note that the returns shown here are all excess returns compared to the Treasury Bill rate rather than actual returns. However, differences between returns would still be the same.

¹³⁹ Of course, this \$100 million figure is not meant to signify what the damages would have been in this case. That, of course, would entail additional factors, including estimates of the prices that shares were actually bought and sold at during the relevant class period.

multi-factor model than the single-factor model, depending upon circumstances, damage estimates could be higher in other cases. Second, although in this case we used the three-factor model that we have used throughout this Article, in a particular damage suit, a custom designed multi-factor model for the particular stock in question could be still more accurate.

CONCLUSION

This Article analyzed model choice for calculating the expected return to a manipulated security, recognizing that the calculation of expected return is tantamount to the measurement of damages. We argued that a multi-factor *ex post* model was the best way to measure damages under Rule 10b-5. *Ex ante* models like the CAPM and the APT may well be suitable for investment decisions, but they are not suitable for damage measurement. Therefore, the most important issue is to determine whether the single-factor market model or a more general multi-factor model is more accurate in assessing damages. Only the Adams et al. study addresses this issue, and that study found that a multi-factor model offers a very significant improvement over a single-factor market model. Therefore, the measurement of damages by means of the single-factor market model or even a market model with an industry index component is likely to be very unreliable. Not only are the resulting estimates of damages unreliable, but the degree of unreliability is not even susceptible to approximation.

While perfect estimation of the effect of general economic conditions on a manipulated security's returns remains the goal in measurement of Rule 10b-5 damages cases, it will probably remain unattainable. The most that one can expect is a reasonable approximation. In the field of capital market theory, new models of asset returns are accepted when they can help explain those returns, and old models are discarded when new models yield further understanding. Our results suggest that a multi-factor model is a superior alternative to the market model and that the multi-factor model, rather than the market model or any other variant of the CAPM, represents the best means of measuring damages in Rule 10b-5 cases.

APPENDIX

This appendix summarizes the empirical results in Adams et al.¹⁴⁰ In that paper those authors examined the relative performance of single- and multi-factor models in explaining the actual returns on portfolios of electric utility firms ranked by decile. In each case the dependent variable is the excess realized return for the portfolio of electric utilities over the Treasury Bill rate on a monthly basis from the beginning of 1960 through the end of 1994. For the single-factor model the independent variables were a constant and the return over the Treasury Bill rate on the value-weighted portfolio of securities on the New York and American Exchanges. The multi-factor model also adds a risk variable which is simply the difference between the yield BAA corporate bonds and the yield on long term Treasury Bonds adjusted to be at a monthly rate. A third factor, the steepness of the term-structure of interest rates, measures the difference between the yield on long term government bonds and Treasury Bills. Once again, this is also adjusted to be at a monthly rate. The independent variables are regressed on the dependent variables. The results of these regressions are shown in Tables 1 and 2 below. Note that for each portfolio, in the multi-factor model the term structure and risk variables are significant at the 0.01 level. The tables show the coefficient estimates for the variables in each model and denote the level of statistical significance for each coefficient.

TABLE 1: SINGLE-FACTOR MODEL

Portfolio	Constant	Market
1	0.0009	0.5544*
2	0.0027	0.5467*
3	0.0018	0.5201*
4	0.0023	0.5443*
5	0.0025	0.6162*
6	0.0021	0.6200*
7	0.0015	0.5971*
8	0.0010	0.5733*
9	0.0009	0.6275*
10	0.0010	0.6050*

*: Coefficient significant at the 0.05 level

**: Coefficient significant at the 0.01 level

¹⁴⁰ See *supra* note 132.

TABLE 2: MULTI-FACTOR MODEL

Portfolio	Constant	Market	Risk	Term-structure
1	0.0008	0.4444**	0.7466**	0.4824**
2	0.0026	0.4124**	0.6256**	0.6006**
3	0.0017	0.3747**	0.6756**	0.6504**
4	0.0022	0.3974**	0.5854**	0.6606**
5	0.0024	0.4752**	0.6340**	0.6315**
6	0.0020	0.4841**	0.5232**	0.6118**
7	0.0014	0.4555**	0.5245**	0.6388**
8	0.0009	0.4204**	0.5233**	0.6912**
9	0.0008	0.4797**	0.6361**	0.6629**
10	0.0009	0.4598**	0.4550**	0.6580**